

# BC Practice Examination 4

## Section I \_\_\_\_\_

### Part A<sup>†</sup>

(See instructions, page 532. Answers are given on page 580.)

1.  $\lim_{x \rightarrow 2} [x]$  (where  $[x]$  is the greatest integer in  $x$ ) is  
(A) 1      (B) 2      (C) 3      (D)  $\infty$       (E) nonexistent

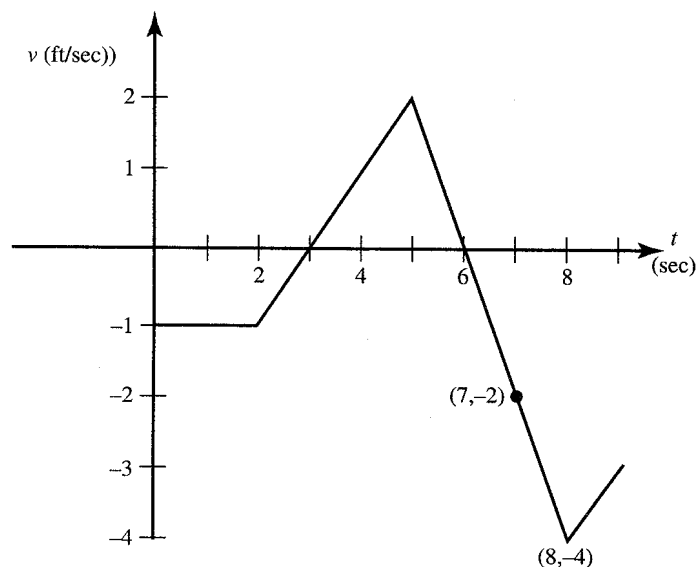
2.  $\lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} + h\right) - 1}{h}$  is  
(A) 1      (B) -1      (C) 0      (D)  $\infty$       (E) none of these

3. The set of all  $x$  for which the power series  $\sum_{n=0}^{\infty} \frac{x^n}{(n+1) \cdot 3^n}$  converges is  
(A)  $\{-3, 3\}$       (B)  $|x| < 3$       (C)  $|x| > 3$       (D)  $-3 \leq x < 3$   
(E)  $-3 < x \leq 3$

4. The equation of the tangent to the curve  $2x^2 - y^4 = 1$  at the point  $(-1, 1)$  is  
(A)  $y = -x$       (B)  $y = 2 - x$       (C)  $4y + 5x + 1 = 0$   
(D)  $x - 2y + 3 = 0$       (E)  $x - 4y + 5 = 0$

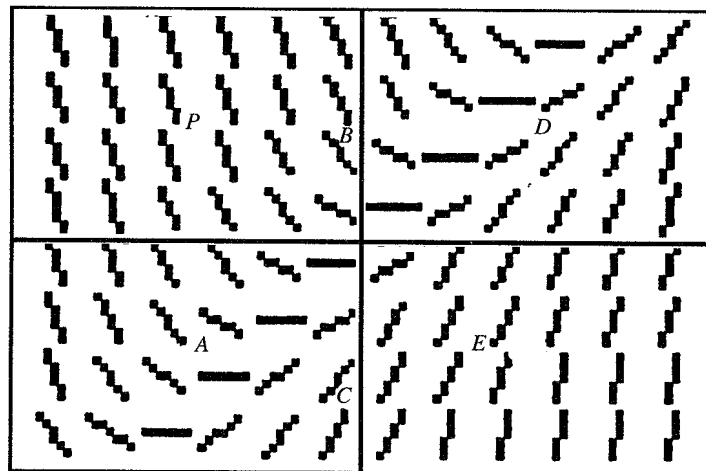
The graph above is for Questions 5 and 6. It shows the velocity of an object during the

<sup>†</sup>Beginning in May 1998, 55 minutes will be allowed for Part A.



interval  $0 \leq t \leq 9$ .

5. The object attains its greatest speed at  $t =$   
 (A) 2      (B) 3      (C) 5      (D) 6      (E) 8
6. The object was at the origin at  $t = 3$ . It returned to the origin  
 (A) at  $t = 5$       (B) at  $t = 6$       (C) during  $6 < t < 7$   
 (D) at  $t = 7$       (E) during  $7 < t < 8$
7. A relative maximum value of the function  $y = \frac{\ln x}{x}$  is  
 (A) 1      (B)  $e$       (C)  $\frac{2}{e}$       (D)  $\frac{1}{e}$       (E) none of these
8. When a series is used to approximate  $\int_0^{0.3} e^{-x^2} dx$ , the value of the integral, to two decimal places, is  
 (A) -0.09      (B) 0.29      (C) 0.35      (D) 0.81      (E) 1.35

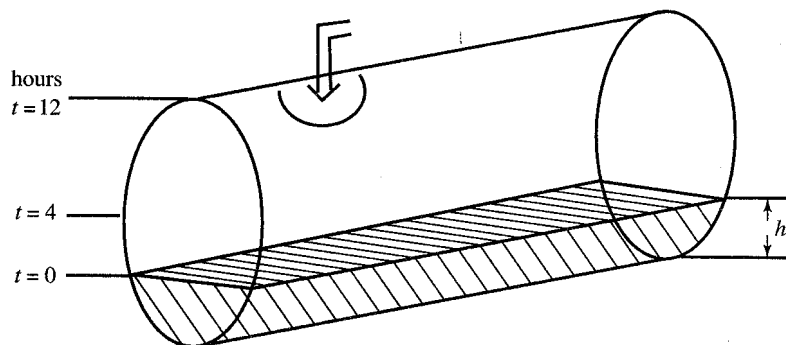


9. A particular solution of the differential equation whose slope field is shown above contains point  $P$ . This solution may also contain which other point ( $A, B, C, D, E$ )?

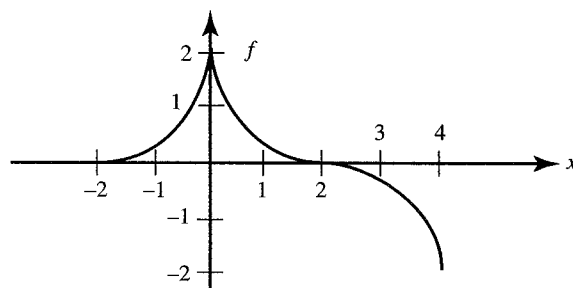
10. Let  $F(x) = \int_5^x \frac{dt}{1-t^2}$ . Which is true?  
 I. The domain of  $F$  is  $x \neq \pm 1$ .    II.  $F(2) > 0$ .    III.  $F$  is concave upward.  
 (A) none    (B) I only    (C) II only    (D) III only  
 (E) II and III only
11. As the tides change, the water-level in a bay varies sinusoidally. At high tide today at 8 A.M., the water-level was 15 feet; at low tide, 6 hours later at 2 P.M. it was 3 feet. How fast, in ft per hr, was the water-level dropping at noon today?  
 (A) 3    (B)  $\frac{\pi\sqrt{3}}{2}$     (C)  $3\sqrt{3}$     (D)  $\pi\sqrt{3}$     (E)  $6\sqrt{3}$
12. Let  $\int_0^x f(t) dt = x \sin \pi x$ . Then  $f(3) =$   
 (A)  $-3\pi$     (B)  $-1$     (C) 0    (D) 1    (E)  $3\pi$
13.  $\int \frac{e^u}{4 + e^{2u}} du$  is equal to  
 (A)  $\ln(4 + e^{2u}) + C$     (B)  $\frac{1}{2} \ln|4 + e^{2u}| + C$     (C)  $\frac{1}{2} \tan^{-1} \frac{e^u}{2} + C$   
 (D)  $\tan^{-1} \frac{e^u}{2} + C$     (E)  $\frac{1}{2} \tan^{-1} \frac{e^{2u}}{2} + C$
14. Given  $f(x) = \log_{10} x$  and  $\log_{10}(102) \approx 2.0086$ , which is closest to  $f'(100)$ ?  
 (A) 0.0043    (B) 0.0086    (C) 0.01    (D) 1.0043    (E) 2
15. If  $G(2) = 5$  and  $G'(x) = \frac{10x}{9 - x^2}$ , then an estimate of  $G(2.2)$  using local linearization is approximately  
 (A) 5.4    (B) 5.5    (C) 5.8    (D) 8.8    (E) 13.8
16. The area bounded by the parabola  $y = x^2$  and the lines  $y = 1$  and  $y = 9$  equals  
 (A) 8    (B)  $\frac{84}{3}$     (C)  $\frac{64}{3}\sqrt{2}$     (D) 32    (E)  $\frac{104}{3}$
17. The first-quadrant region bounded by  $y = \frac{1}{\sqrt{x}}$ ,  $y = 0$ ,  $x = q$  ( $0 < q < 1$ ), and  $x = 1$  is rotated about the  $x$ -axis. The volume obtained as  $q \rightarrow 0^+$  equals  
 (A)  $\frac{2\pi}{3}$     (B)  $\frac{4\pi}{3}$     (C)  $2\pi$     (D)  $4\pi$     (E) none of these
18. A curve is given parametrically by the equations  

$$x = 3 - 2 \sin t \text{ and } y = 2 \cos t - 1.$$
 The length of the arc from  $t = 0$  to  $t = \pi$  is  
 (A)  $\frac{\pi}{2}$     (B)  $\pi$     (C)  $2 + \pi$     (D)  $2\pi$     (E)  $4\pi$

19. Suppose the function  $f$  is both increasing and concave up on  $a \leq x \leq b$ . Then, using the same number of subdivisions, and with  $L$ ,  $R$ ,  $M$ , and  $T$  denoting respectively Left, Right, Midpoint, and Trapezoid sums, it follows that  
 (A)  $R \leq T \leq M \leq L$       (B)  $L \leq T \leq M \leq R$       (C)  $R \leq M \leq T \leq L$   
 (D)  $L \leq M \leq T \leq R$       (E) none of these
20. Which of the following statements about the graph of  $y = \frac{x^2}{x-2}$  are true?  
 I. The graph has no horizontal asymptote.  
 II. The line  $x = 2$  is a vertical asymptote.  
 III. The line  $y = x + 2$  is an oblique asymptote.  
 (A) I only      (B) II only      (C) I and II only  
 (D) I and III only      (E) all three statements
21. The only function that does not satisfy the Mean Value Theorem on the interval specified is  
 (A)  $f(x) = x^2 - 2x$  on  $[-3, 1]$ .  
 (B)  $f(x) = \frac{1}{x}$  on  $[1, 3]$ .  
 (C)  $f(x) = \frac{x^3}{3} - \frac{x^2}{2} + x$  on  $[-1, 2]$ .  
 (D)  $f(x) = x + \frac{1}{x}$  on  $[-1, 1]$ .  
 (E)  $f(x) = x^{2/3}$  on  $\left[\frac{1}{2}, \frac{3}{2}\right]$ .
22.  $\int_0^1 x^2 e^x dx =$   
 (A)  $-3e - 1$       (B)  $-e$       (C)  $e - 2$       (D)  $3e$       (E)  $4e - 1$



23. A cylindrical tank is partially full of water at time  $t = 0$ , when more water begins flowing in at a constant rate. The tank becomes half full when  $t = 4$ , and is completely full when  $t = 12$ . Let  $h$  represent the height of the water at time  $t$ . During which interval is  $\frac{dh}{dt}$  increasing?  
 (A) never      (B)  $0 < t < 4$       (C)  $0 < t < 8$       (D)  $0 < t < 12$   
 (E)  $4 < t < 12$



24. The graph of function  $f$  consists of 3 quarter-circles.
- Which of the following is equivalent to  $\int_0^2 f(x) dx$ ?
- I.  $\frac{1}{2} \int_{-2}^2 f(x) dx$       II.  $\int_4^2 f(x) dx$       III.  $\frac{1}{2} \int_0^4 f(x) dx$
- (A) I only      (B) II only      (C) III only      (D) I and II only  
(E) all of these
25. The base of a solid is the first-quadrant region bounded by  $y = \sqrt[4]{4 - 2x}$ , and each cross-section perpendicular to the  $x$ -axis is a semicircle with a diameter in the  $xy$ -plane. The volume of the solid is
- (A)  $\frac{\pi}{2} \int_0^2 \sqrt{4 - 2x} dx$       (B)  $\frac{\pi}{8} \int_0^2 \sqrt{4 - 2x} dx$       (C)  $\frac{\pi}{8} \int_{-2}^2 \sqrt{4 - 2x} dx$
- (D)  $\frac{\pi}{4} \int_0^{\sqrt{2}} (4 - y^4)^2 dy$       (E)  $\frac{\pi}{8} \int_0^{\sqrt[4]{4}} (4 - y^4)^2 dy$
26. The average value of  $f(x) = 3 + |x|$  on the interval  $[-2, 4]$  is
- (A)  $2\frac{2}{3}$       (B)  $3\frac{1}{3}$       (C)  $4\frac{2}{3}$       (D)  $5\frac{1}{3}$       (E) 6
27. The area inside the circle  $r = 3 \sin \theta$  and outside the cardioid  $r = 1 + \sin \theta$  is given by
- (A)  $\int_{\pi/6}^{\pi/2} [9 \sin^2 \theta - (1 + \sin \theta)^2] d\theta$       (B)  $\int_{\pi/6}^{\pi/2} (2 \sin \theta - 1)^2 d\theta$
- (C)  $\frac{1}{2} \int_{\pi/6}^{5\pi/6} (8 \sin^2 \theta - 1) d\theta$       (D)  $\frac{9\pi}{4} - \frac{1}{2} \int_{\pi/6}^{5\pi/6} (1 + \sin \theta)^2 d\theta$
- (E) none of these

28. Let

$$f(x) = \begin{cases} \frac{x^2 - 36}{x - 6} & \text{if } x \neq 6, \\ 12 & \text{if } x = 6. \end{cases}$$

Which of the following statements is (are) true?

- I.  $f$  is defined at  $x = 6$ .  
 II.  $\lim_{x \rightarrow 6} f(x)$  exists.  
 III.  $f$  is continuous at  $x = 6$ .
- (A) I only      (B) II only      (C) I and II only      (D) I, II, and III  
 (E) none of the statements

**Part B<sup>†</sup>**

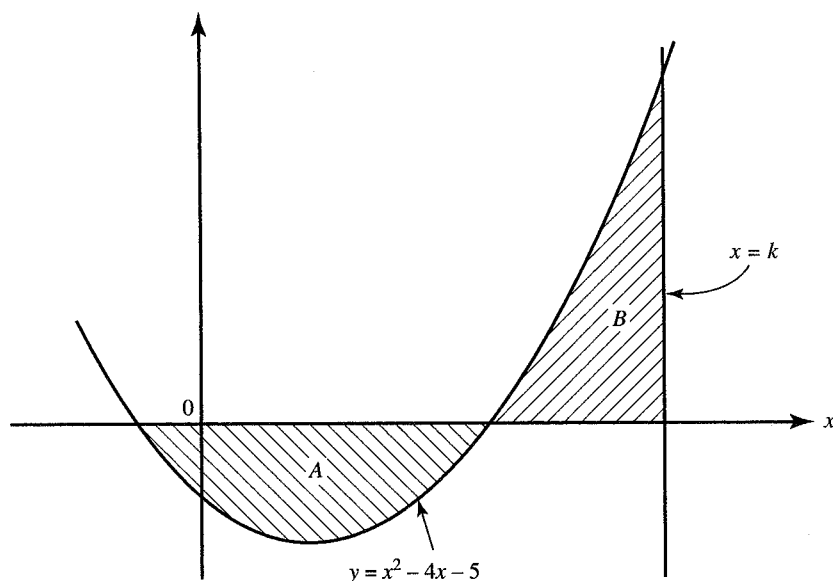
See instructions, page 536.

29. Two objects in motion from  $t = 0$  to  $t = 3$  seconds have positions,  $x_1(t) = \cos(t^2 + 1)$  and  $x_2(t) = \frac{e^t}{2t}$  respectively. How many times during the three seconds do the objects have the same velocity?
- (A) 0      (B) 1      (C) 2      (D) 3      (E) 4

$x$	-1	0	1	2	3
$f''(x)$	-4	-1	2	5	8

30. The table above shows values of  $f''(x)$  for various values of  $x$ . The function  $f$  could be
- (A) a linear function      (B) a quadratic function      (C) a cubic function  
 (D) a fourth-degree function      (E) an exponential function
31. Where, in the first quadrant, does the rose  $r = \sin 3\theta$  have a vertical tangent?
- (A) nowhere      (B)  $\theta = 0.39$       (C)  $\theta = 0.47$   
 (D)  $\theta = 0.52$       (E)  $\theta = 0.60$
32. A cup of coffee placed on a table cools at a rate of  $\frac{dH}{dt} = -0.05(H - 70)$  degrees per minute, where  $H$  represents its temperature and  $t$  is time in minutes. If the coffee was at  $120^\circ$  F initially, what will its temperature be 10 minutes later?
- (A)  $73^\circ$  F      (B)  $95^\circ$  F      (C)  $100^\circ$  F      (D)  $118^\circ$  F      (E)  $143^\circ$  F
33. An investment of \$4000 grows at the rate of  $320e^{0.08t}$  dollars per year after  $t$  years. Its value after 10 years is approximately
- (A) \$4902      (B) \$8902      (C) \$7122      (D) \$12,902  
 (E) none of these

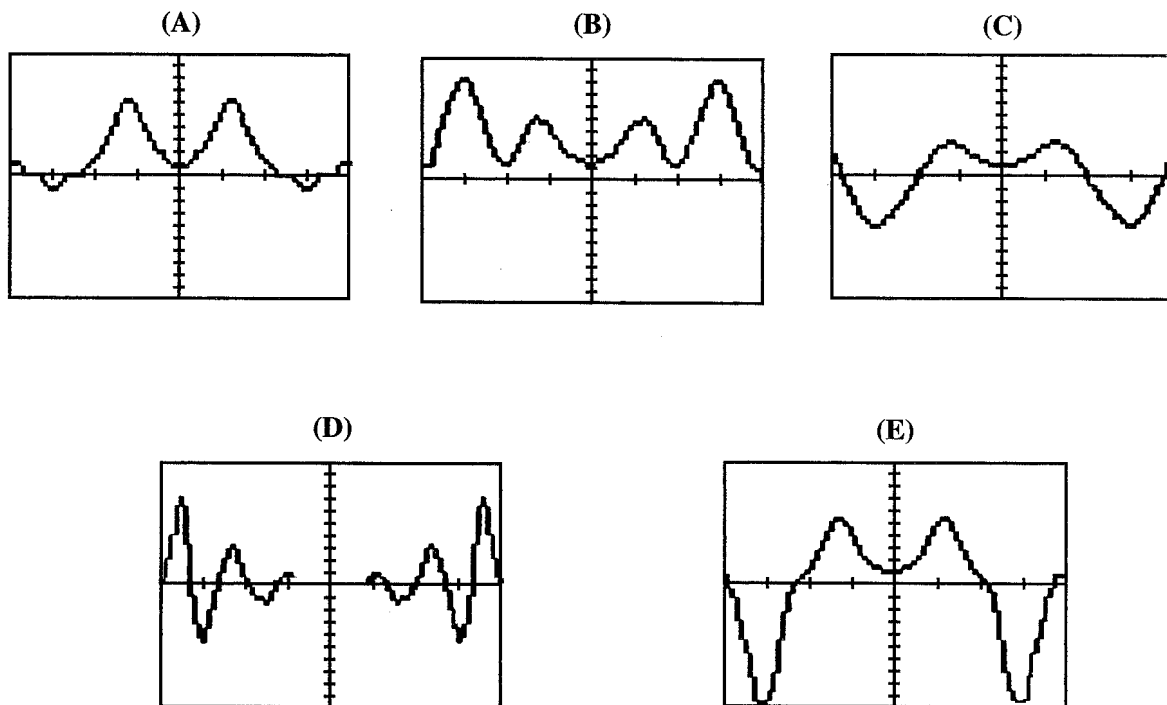
<sup>†</sup>Beginning in May 1998, 50 minutes will be allowed for Part B.



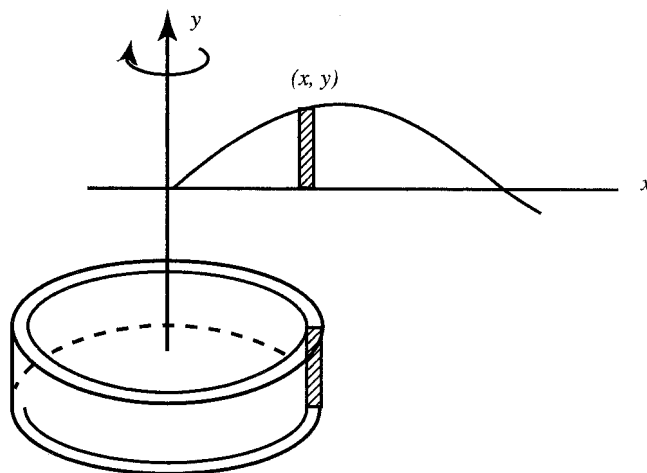
(This figure is not drawn to scale.)

34. The sketch shows the graphs of  $f(x) = x^2 - 4x - 5$  and the line  $x = k$ . The regions labeled  $A$  and  $B$  have equal areas if  $k =$   
 (A) 5      (B) 7.766      (C) 7.899      (D) 8      (E) 11
35. The  $n$ th term of the Taylor series expansion about  $x = 0$  of the function  $f(x) = \frac{1}{1 + 2x}$  is  
 (A)  $(2x)^n$       (B)  $2x^{n-1}$       (C)  $\left(\frac{x}{2}\right)^{n-1}$       (D)  $(-1)^{n-1}(2x)^{n-1}$   
 (E)  $(-1)^n (2x)^{n-1}$
36. When the method of partial fractions is used to decompose  $\frac{2x^2 - x + 4}{x^3 - 3x^2 + 2x}$ , one of the fractions obtained is  
 (A)  $-\frac{5}{x-1}$       (B)  $-\frac{2}{x-1}$       (C)  $\frac{1}{x-1}$       (D)  $\frac{2}{x-1}$       (E)  $\frac{5}{x-1}$
37. An object in motion in the plane has acceleration vector  $\mathbf{a}(t) = (\sin t, e^{-t})$  for  $0 \leq t \leq 5$ . It is at rest when  $t = 0$ . What is the maximum speed it attains?  
 (A) 1.022      (B) 1.414      (C) 2.217      (D) 2.958      (E) 3.162
38. If we replace  $\sqrt{x-2}$  by  $u$ , then  $\int_3^6 \frac{\sqrt{x-2}}{x} dx$  is equivalent to  
 (A)  $\int_1^2 \frac{u du}{u^2 + 2}$       (B)  $2 \int_1^2 \frac{u^2 du}{u^2 + 2}$       (C)  $\int_3^6 \frac{2u^2}{u^2 + 2} du$       (D)  $\int_3^6 \frac{u du}{u^2 + 2}$   
 (E)  $\frac{1}{2} \int_1^2 \frac{u^2}{u^2 + 2} du$

39. Which of the following could be the graph of  $y = \sum_{n=0}^9 \frac{(x \sin x)^n}{n!}$  on  $-2\pi \leq x \leq 2\pi$ ?

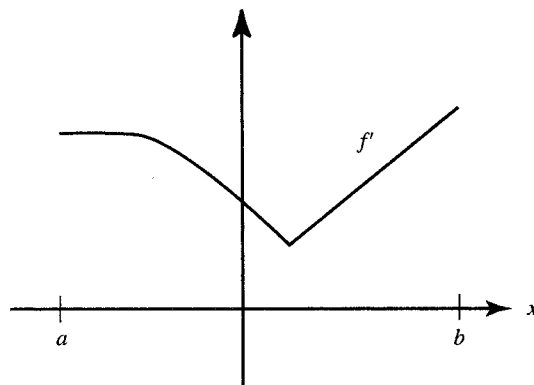


40. A particle moves along a line with acceleration  $a = 6t$ . If, when  $t = 0$ ,  $v = 1$ , then the total distance traveled between  $t = 0$  and  $t = 3$  equals  
 (A) 30      (B) 28      (C) 27      (D) 26      (E) none of these
41. The first arch of  $y = \sin x$  is rotated around the  $y$ -axis. The element shown forms a cylindrical shell whose volume is the product of its surface area and its thickness. The total volume of solid formed (the sum of the volumes of all such shells) is given by  
 (A)  $\pi \int_0^\pi \sin^2 x \, dx$       (B)  $\pi \int_0^\pi x \sin x \, dx$       (C)  $2\pi \int_0^\pi x \sin x \, dx$   
 (D)  $2\pi \int_0^\pi \sin x \, dx$       (E) none of these





42. Suppose  $f(3) = 2$ ,  $f'(3) = 5$ , and  $f''(3) = -2$ . Then  $\frac{d^2}{dx^2}(f^2(x))$  at  $x = 3$  is equal to  
 (A)  $-20$       (B)  $10$       (C)  $20$       (D)  $38$       (E)  $42$
43. Which statement is true?  
 (A) If  $f(x)$  is continuous at  $x = c$ , then  $f'(c)$  exists.  
 (B) If  $f'(c) = 0$ , then  $f$  has a local maximum or minimum at  $(c, f(c))$ .  
 (C) If  $f''(c) = 0$ , then  $f$  has an inflection point at  $(c, f(c))$ .  
 (D) If  $f$  is differentiable at  $x = c$ , then  $f$  is continuous at  $x = c$ .  
 (E) If  $f$  is continuous on  $(a, b)$ , then  $f$  maintains a maximum value on  $(a, b)$ .



44. The graph of  $f'$  is shown above. Which statements about  $f$  must be true for  $a < x < b$ ?  
 I.  $f$  is increasing      II.  $f$  is continuous      III.  $f$  is differentiable  
 (A) I only      (B) II only      (C) I and II only      (D) I and III only  
 (E) all three
45. After a bomb explodes, pieces can be found scattered around the center of the blast. The density of bomb fragments lying  $x$  meters from ground zero is given by  $N(x) = \frac{2x}{1 + x^{3/2}}$  fragments per square meter. How many fragments will be found within 20 meters of the point where the bomb exploded?  
 (A) 13      (B) 278      (C) 556      (D) 712      (E) 4383

# Answers to BC Practice Examination 4: Section I

1. E	10. E	19. D	28. D	37. C
2. C	11. B	20. E	29. E	38. B
3. D	12. A	21. D	30. C	39. A
4. A	13. C	22. C	31. C	40. A
5. E	14. A	23. E	32. C	41. C
6. E	15. C	24. D	33. B	42. E
7. D	16. E	25. B	34. D	43. D
8. B	17. E	26. C	35. D	44. E
9. D	18. D	27. A	36. A	45. D

Some of the questions in BC Practice Examination 4 are the same as those bearing the same number in AB Practice Examination 4. Whenever this is so, the explanation is not given below; instead, it will be found in the Answers to AB Practice Examination 4, on pages 523 to 531.

### Part A

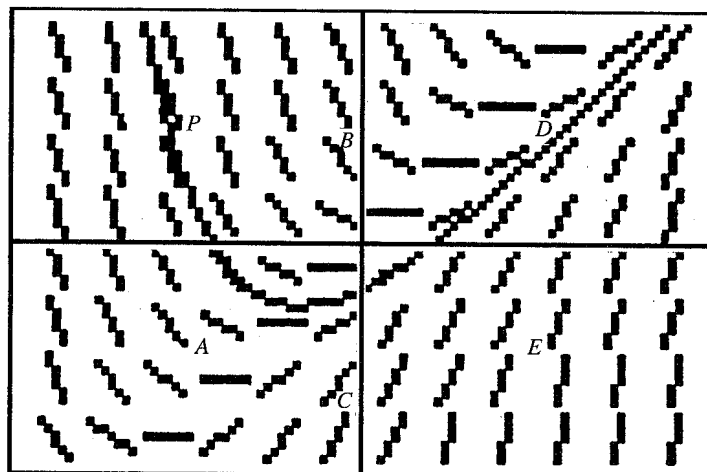
3. D. Use the Ratio Test:

$$\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{(n+2) \cdot 3^{n+1}} \cdot \frac{(n+1) \cdot 3^n}{x^n} \right| = \lim_{n \rightarrow \infty} \frac{n+1}{n+2} \cdot \frac{1}{3} |x| = \frac{|x|}{3},$$

which is less than 1 if  $-3 < x < 3$ . When  $x = -3$ , we get the convergent alternating harmonic series.

8. B. Since  $e^x \approx 1 + x$ ,  $e^{-x^2} \approx 1 - x^2$ . So

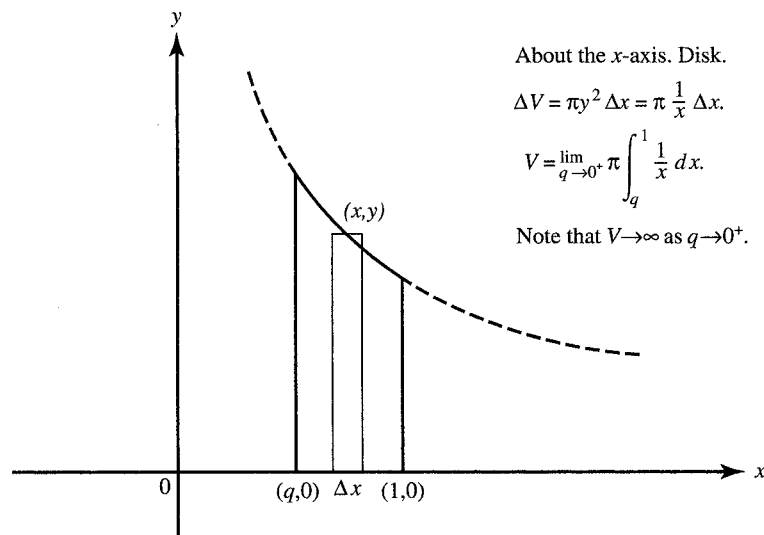
$$\int_0^{0.3} (1 - x^2) dx = x - \frac{x^3}{3} \Big|_0^{0.3} = 0.3 - \frac{0.027}{3}$$



9. D. The slopefield suggests the curve shown above as a particular solution.

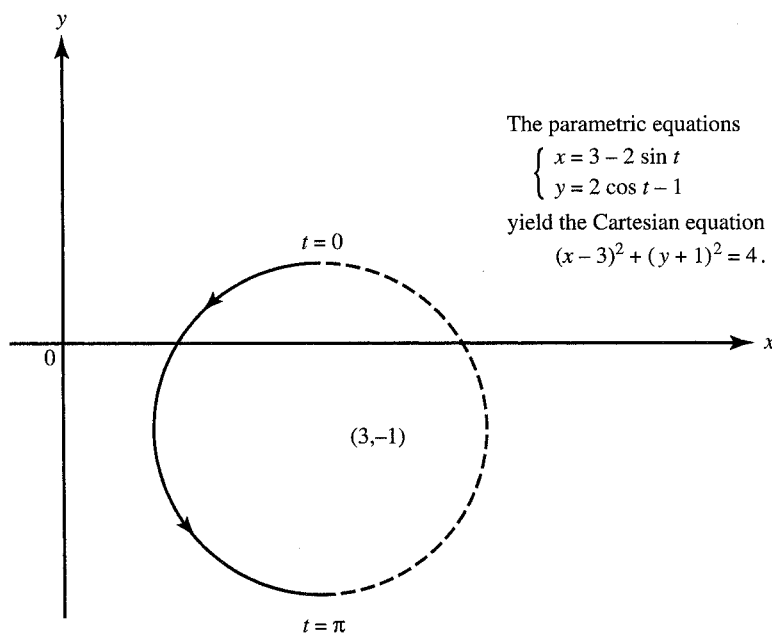
12. A.  $f(x) = \frac{d}{dx}(x \sin \pi x) = \pi x \cos \pi x + \sin \pi x.$

17. E.



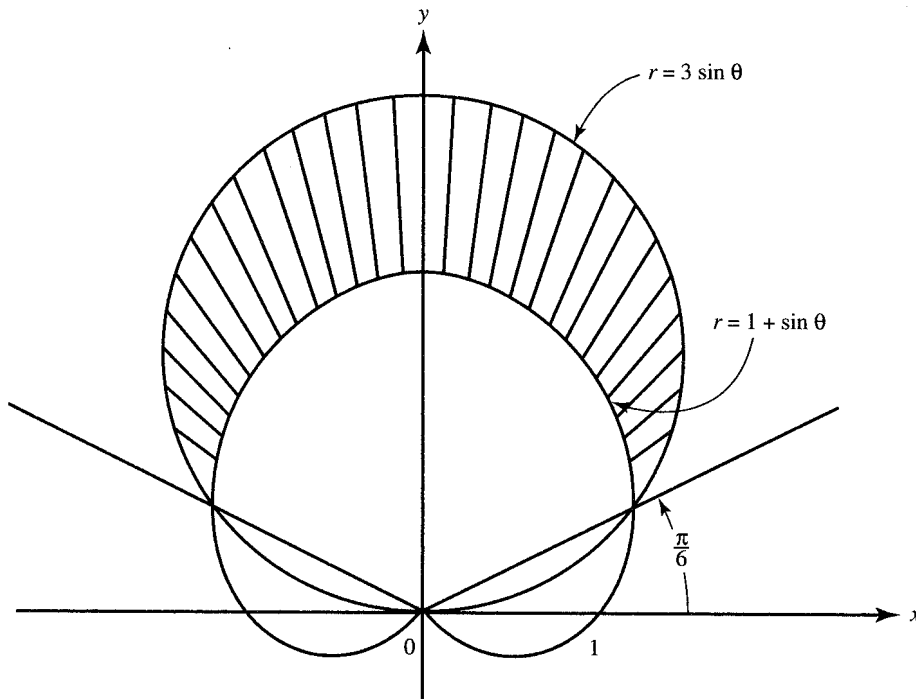
18. D. See the figure, which shows that we seek the length of a semicircle of radius 2 here. The answer can, of course, be found by using the formula for arc length:

$$s = \int_0^\pi \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$



20. E.  $\lim_{x \rightarrow \infty} f(x) = \infty; \lim_{x \rightarrow 2^-} f(x) = -\infty; y = x + 2 + \frac{4}{x - 2}.$

22. C. Using parts twice yields the antiderivative  $x^2e^x - 2xe^x + 2e^x$ .
27. A. The required area is lined in the figure.



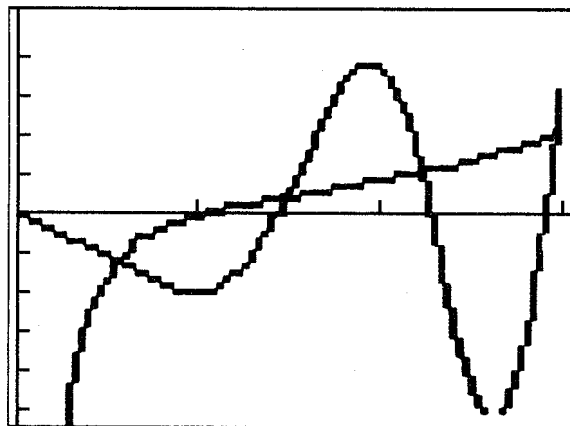
28. D. Note that  $f(x) = x + 6$  if  $x \neq 6$ , that  $f(6) = 12$ , and that  $\lim_{x \rightarrow 6} f(x) = 12$ . So  $f$  is continuous at  $x = 6$ , which implies the truth of I and II.

**Part B**

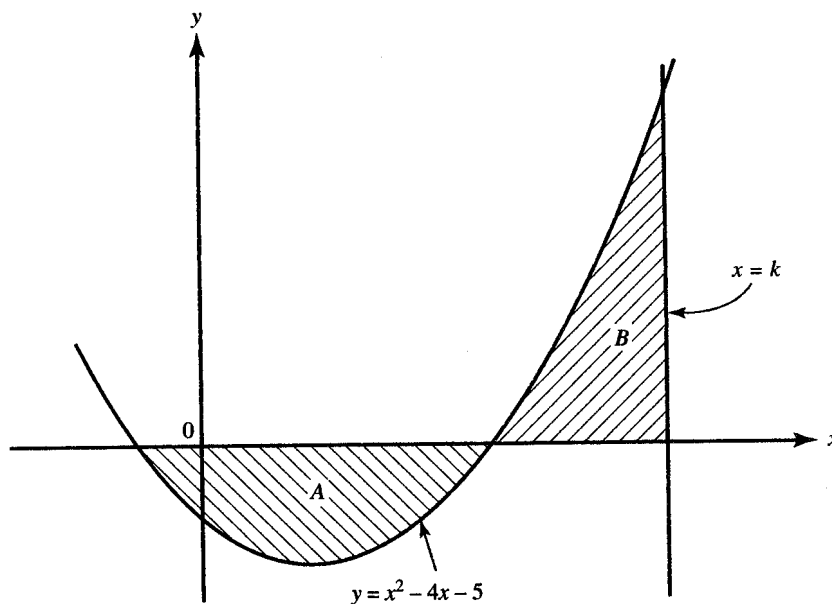
29. E. The velocity functions are,

$$v_1 = -2t \sin(t^2 + 1) \text{ and } v_2 = \frac{2t(e^t) - 2e^t}{(2t)^2} = \frac{e^t(t - 1)}{2t^2}$$

Let  $X$  be  $t$ , let  $Y_1 = -2X \sin(X^2 + 1)$  and  $Y_2 = (e^X)(X - 1) / 2X^2$ . Graph  $Y_1$  and  $Y_2$  in  $[0, 3] \times [-5, 5]$ . The graphs intersect four times during the first three seconds.



30. C. Changes in values of  $f''$  show that  $f'''$  is constant.
31. C. Expressed parametrically,  $x = \sin 3\theta \cos \theta$ ,  $y = \sin 3\theta \sin \theta$ . Where  $\frac{dx}{d\theta} = -\sin 3\theta \sin \theta + 3 \cos 3\theta \cos \theta = 0$ ,  $\frac{dy}{dx}$  is undefined. Use the [solve] option to find  $\theta$ .
34. D.



(This figure is not drawn to scale.)

The roots of  $f(x) = x^2 - 4x - 5 = (x - 5)(x + 1)$  are  $x = -1$  and  $5$ . Since areas  $A$  and  $B$  are equal, therefore  $\int_{-1}^k f(x)(dx) = 0$ . Thus,

$$\begin{aligned} \left(\frac{x^3}{3} - 2x^2 - 5x\right)\Big|_{-1}^k &= \left(\frac{k^3}{3} - 2k^2 - 5k\right) - \left(-\frac{1}{3} - 2 + 5\right) \\ &= \frac{k^3}{3} - 2k^2 - 5k - \frac{8}{3} = 0 \end{aligned}$$

Using 6 as a guess for the root  $k$ , enter

$$\text{solve}(X^3/3 - 2X^2 - 5X - 8/3, X, 6)$$

This yields  $k = 8$ .

35. D. The series is  $1 - 2x + 4x^2 - 8x^3 + 16x^4 - \dots$

36. A. Assume that

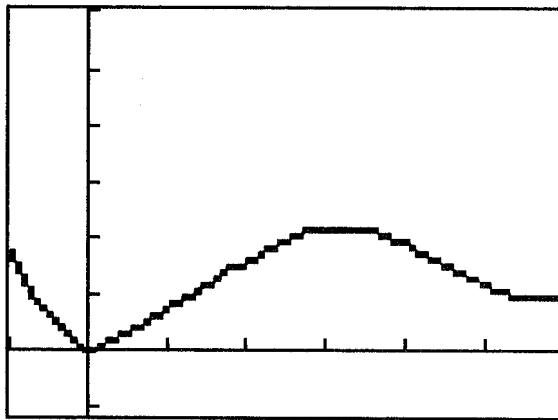
$$\frac{2x^2 - x + 4}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}.$$

Then

$$2x^2 - x + 4 = A(x-1)(x-2) + Bx(x-2) + Cx(x-1).$$

Since we are looking for  $B$ , we let  $x = 1$ :

$$2(1) - 1 + 4 = 0 + B(-1) + 0; B = -5.$$



37. C. We are given that  $\mathbf{a}(t) = (\sin t, e^{-t})$ . An antiderivative is  $\mathbf{v}(t) = (-\cos t + c_1, -e^{-t} + c_2)$ . Since  $\mathbf{v}(0) = (0,0)$ , the constants are  $c_1 = c_2 = 1$ . The object's speed is

$$|\mathbf{v}(t)| = \sqrt{(-\cos t + 1)^2 + (-e^{-t} + 1)^2}$$

Let  $Y_1 = \sqrt{((-\cos X + 1)^2 + (-e^{-X} + 1)^2)}$  and graph  $Y_1$  in  $[-1, 6] \times [-1, 6]$ . The calculator's [maximum] option shows on the screen that the greatest speed is  $Y = 2.217$ . (Of course,  $Y = Y_1$ .)

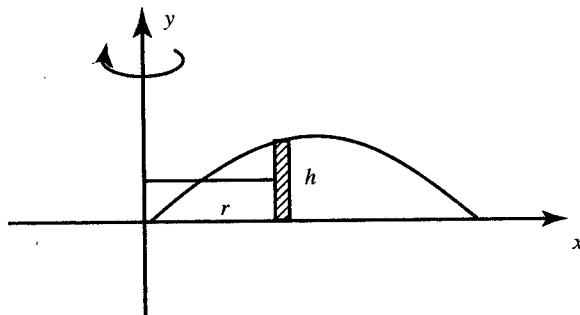
39. A. On the TI-82 we use the [sum] operation on the LIST MATH menu followed by the [seq] operation on the LIST OPS menu to key in

$$Y_1 = \text{sum seq}((X \sin X)^N / N!, N, 0, 9, 1)$$

The entry before the first comma above is the expression to be summed;  $N$  is the variable (replacing  $n$  in the question); the sum goes from  $N = 0$  to  $N = 9$  by increments of 1. Graph  $Y_1$  in  $[-2\pi, 2\pi] \times [-10, 10]$ . The graph that appears is the one in choice A.

Note, in the question, that we've defined the distance between tick marks on the  $x$ -axis in Figure A to be  $\pi/2$  (that is, on the calculator screen,  $X\text{scl} = \pi/2$ ).

40. A. Since  $a = \frac{dv}{dt} = 6t$ ,  $v = 3t^2 + C$ , and since  $v(0) = 1$ ,  $C = 1$ . Then  $v = \frac{ds}{dt} = 3t^2 + 1$  yields  $s = t^3 + t + C'$ , and we can let  $s(0) = 0$ . Then we want  $s(3)$ .



41. C.  $\Delta V = (\text{lateral surface area})(\text{thickness}) = (2\pi rh)(\Delta x) = 2\pi xy \cdot \Delta x$ .

$$V = 2\pi \int_0^{\pi} x \sin x \, dx$$

42. E.  $\frac{d}{dx}(f^2(x)) = 2f(x)f'(x)$ ,

$$\frac{d^2}{dx^2}(f^2(x)) = 2[f(x)f''(x) + f'(x)f'(x)]$$

$$= 2[ff'' + (f')^2]$$

At  $x = 3$ ; the answer is  $2[2(-2) + 5^2] = 42$ .

43. D. Counterexamples are, respectively: for (A),  $f(x) = |x|$ ,  $c = 0$ ; for (B),  $f(x) = x^3$ ,  $c = 0$ ; for (C),  $f(x) = x^4$ ,  $c = 0$ ; for (E),  $f(x) = x^2$  on  $(-1, 1)$ .