BC Practice Examination 3

Section I

Part A†

(See instructions, page 532. Answers are given on page 565.)

1. A function $f(x)$ equals $\dfrac{x^2 - x}{x - 1}$ for all x except $x = 1$. In order that the function be continuous at $x = 1$, the value of $f(1)$ must be
   (A) 0 **(B)** 1 **(C)** 2 **(D)** ∞ **(E)** none of these

2. $\displaystyle\lim_{x\to 0} \frac{\sin^2 \frac{x}{2}}{x^2}$ is
   (A) 4 **(B)** 0 **(C)** $\frac{1}{4}$ **(D)** 2 **(E)** nonexistent

3. The first four terms of the Taylor series about $x = 0$ of $\sqrt{1 + x}$ are
   (A) $1 - \frac{x}{2} + \frac{x^2}{4 \cdot 2} - \frac{3x^3}{8 \cdot 6}$ **(B)** $x + \frac{x^2}{2} + \frac{x^3}{8} + \frac{x^4}{48}$
   (C) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$ **(D)** $1 + \frac{x}{4} - \frac{x^2}{24} + \frac{x^3}{32}$ **(E)** $-1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$

4. Using the local linearization of $f(x) = \sqrt{9 + \sin(2x)}$ near 0, an estimate of $f(0.06)$ is
   (A) 0.02 **(B)** 2.98 **(C)** 3.01 **(D)** 3.02 **(E)** 3.03

5. Air is escaping from a balloon at a rate of $R(t) = \dfrac{60}{1 + t^2}$ ft^3/min, where t is measured in minutes. How much (in ft^3) escapes during the first minute?
   (A) 15 **(B)** 15π **(C)** 30 **(D)** 30π **(E)** $30 \ln 2$

6. The motion of a particle in a plane is given by the pair of equations $x = e^t \cos t$, $y = e^t \sin t$. The magnitude of its acceleration at any time t equals
   (A) $\sqrt{x^2 + y^2}$ **(B)** $2e^t \sqrt{\cos 2t}$ **(C)** $2e^t$ **(D)** e^t **(E)** $2e^{2t}$

†Beginning in May 1998, 55 minutes will be allowed for Part A.

7. By differentiating term-by-term the series

$$(x-1)+\frac{(x-1)^2}{4}+\frac{(x-1)^3}{9}+\frac{(x-1)^4}{16}+\cdots$$

the interval of convergence obtained is

(A) $0 \leqq x \leqq 2$ **(B)** $0 \leqq x < 2$ **(C)** $0 < x \leqq 2$ **(D)** $0 < x < 2$
(E) only $x = 1$

8. A point moves along the curve $y = x^2 + 1$ so that the x-coordinate is increasing at the constant rate of $\frac{3}{2}$ units per second. The rate, in units per second, at which the distance from the origin is changing when the point has coordinates (1, 2) is equal to

(A) $\frac{7\sqrt{5}}{10}$ **(B)** $\frac{3\sqrt{5}}{2}$ **(C)** $3\sqrt{5}$ **(D)** $\frac{15}{2}$ **(E)** $\sqrt{5}$

9. $\lim_{h\to 0} \frac{\sqrt{25+h}-5}{h}$

(A) $= 0$ **(B)** $= \frac{1}{10}$ **(C)** $= 1$ **(D)** $= 10$ **(E)** does not exist

10. $\int_{\pi/4}^{\pi/3} \sec^2 x \tan^2 x \, dx$ equals

(A) 5 **(B)** $\sqrt{3} - 1$ **(C)** $\frac{8}{3} - \frac{2\sqrt{2}}{3}$ **(D)** $\sqrt{3}$ **(E)** $\sqrt{3} - \frac{1}{3}$

11. $\int_1^e \ln x \, dx$ equals

(A) $\frac{1}{2}$ **(B)** $e - 1$ **(C)** $e + 1$ **(D)** 1 **(E)** -1

12. $\int \frac{(y-1)^2}{2y} dy$ equals

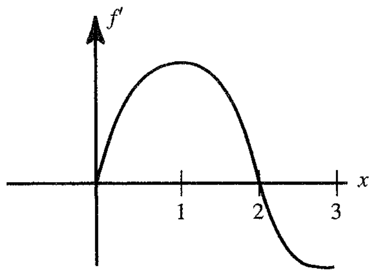
(A) $\frac{y^2}{4} - y + \frac{1}{2}\ln|y| + C$ **(B)** $y^2 - y + \ln|2y| + C$

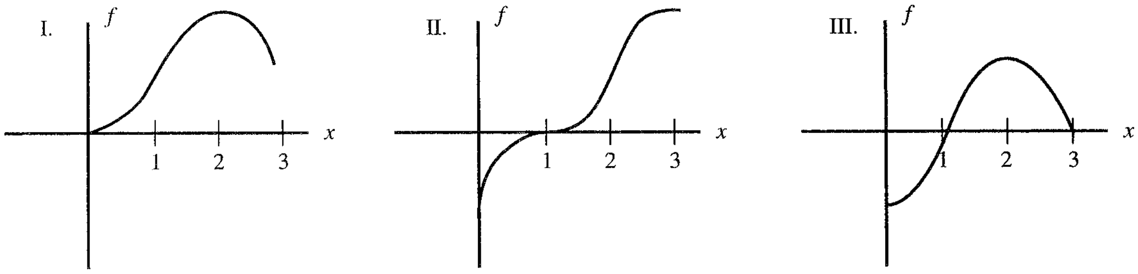
(C) $y^2 - 4y + \frac{1}{2}\ln|2y| + C$ **(D)** $\frac{(y-1)^3}{3y^2} + C$ **(E)** $\frac{1}{2} - \frac{1}{2y^2} + C$

13. Which infinite series converges?

I. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$ III. $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+1}$

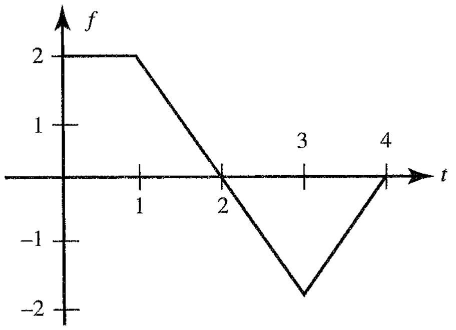
(A) I only **(B)** II only **(C)** III only **(D)** I and III
(E) none of these





14. Given f' as graphed, which could be a graph of f?
(A) I only **(B)** II only **(C)** III only **(D)** I and III
(E) none of these

15. Women have recently begun competing in marathons. At first their times for the 26-mile event dropped rapidly, but of late the times have been declining at a much slower rate. Let $M(t)$ be the curve which best represents winning marathon times in year t. Which of the following is negative?
I. $M(t)$ II. $M'(t)$ III. $M''(t)$
(A) I only **(B)** II only **(C)** III only
(D) II and III **(E)** none of these

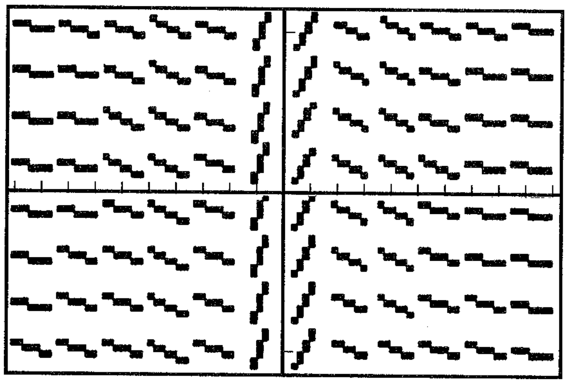


16. The graph of f is above. Let $G(x) = \int_0^x f(t)\,dt$ and $H(x) = \int_2^x f(t)\,dt$. Which of the following is true?
(A) $G(x) = H(x)$ **(B)** $G'(x) = H'(x+2)$
(C) $G(x) = H(x+2)$ **(D)** $G(x) = H(x) - 2$ **(E)** $G(x) = H(x) + 3$

17. Which one of the following improper integrals converges?

(A) $\int_{-1}^{1} \frac{dx}{(x+1)^2}$ **(B)** $\int_{1}^{\infty} \frac{dx}{\sqrt{x}}$ **(C)** $\int_{0}^{\infty} \frac{dx}{(x^2+1)}$ **(D)** $\int_{1}^{3} \frac{dx}{(2-x)^3}$

(E) none of these



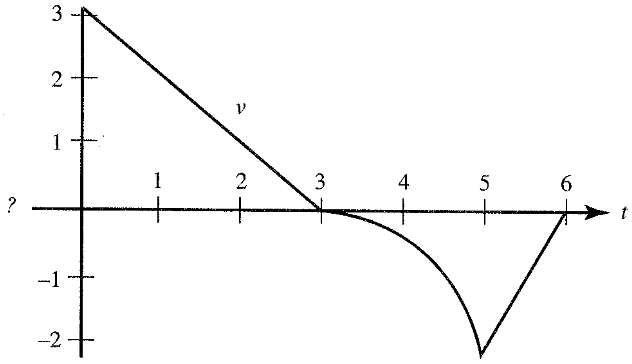
18. Which function could be a particular solution of the differential equation whose slope field is shown above?

(A) $y = x^3$ **(B)** $y = \frac{2x}{x^2+1}$ **(C)** $y = \frac{x^2}{x^2+1}$ **(D)** $y = \sin x$

(E) $y = e^{-x^2}$

19. A particular solution of the differential equation $\frac{dy}{dx} = x + y$ passes through the point (2,1). Using Euler's method with $\Delta x = 0.1$, estimate its y-value at $x = 2.2$.

(A) 0.34 **(B)** 1.30 **(C)** 1.34 **(D)** 1.60 **(E)** 1.64



The graph shown, consisting of two line segments and a quarter circle, is for Questions 20 and 21. It shows the velocity of an object during a 6-second interval.

20. For how many values of t is the acceleration undefined?

(A) none **(B)** one **(C)** two **(D)** three **(E)** four

21. During what time interval is the speed increasing?

(A) $0 < t < 3$ **(B)** $3 < t < 5$ **(C)** $3 < t < 6$ **(D)** $5 < t < 6$

(E) never

22. If $\frac{dy}{dx} = \frac{y}{x}$ $(x > 0, y > 0)$ and $y = 3$ when $x = 1$, then
(A) $x^2 + y^2 = 10$ **(B)** $y = x + \ln 3$ **(C)** $y^2 - x^2 = 8$
(D) $y = 3x$ **(E)** $y^2 - 3x^2 = 6$

23. A solid is cut out of a sphere of radius 2 by two parallel planes each 1 unit from the center. The volume of this solid is
(A) 8π **(B)** $\frac{32\pi}{3}$ **(C)** $\frac{25\pi}{3}$ **(D)** $\frac{22\pi}{3}$ **(E)** $\frac{20\pi}{3}$

24. The length of the arc of $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$ from $x = 1$ to $x = 4$ is
(A) $\frac{15}{2} + \ln 2$ **(B)** $\frac{1}{2}(15 + \ln 2)$ **(C)** $\frac{1}{2}(17 + \ln 2)$ **(D)** $3\frac{15}{64}$
(E) $\frac{45}{16}$

25. Let $f(x) = x^5 + 3x - 2$, and let f^{-1} denote the inverse of f. Then $(f^{-1})'(2)$ equals
(A) $\frac{1}{83}$ **(B)** $\frac{1}{8}$ **(C)** 1 **(D)** 8 **(E)** 83

26. The curve with parametric equations $x = \sqrt{t - 2}$ and $y = \sqrt{6 - t}$ is
(A) part of a circle **(B)** a parabola **(C)** a straight line
(D) part of a hyperbola **(E)** none of these

27. Which of the following statements are true about the graph of $y = \ln(4 + x^2)$?
I. It is symmetric to the y-axis.
II. It has a local minimum at $x = 0$.
III. It has inflection points at $x = \pm 2$.

(A) I only **(B)** II only **(C)** III only **(D)** I and II only
(E) I, II, and III

28. $\int_1^2 \frac{dx}{\sqrt{4 - x^2}}$ is
(A) $-\frac{\pi}{3}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{\pi}{3}$ **(E)** nonexistent

Part B†

(See instructions, page 536.)

29. The area bounded by the curve $x = 3y - y^2$ and the line $x = -y$ is represented by
(A) $\int_0^4 (2y - y^2)\,dy$ **(B)** $\int_0^4 (4y - y^2)\,dy$ **(C)** $\int_0^3 (3y - y^2)\,dy + \int_0^4 y\,dy$

(D) $\int_0^3 (y^2 - 4y)\,dy$ **(E)** $\int_0^3 (2y - y^2)\,dy$

†Beginning in May 1998, 50 minutes will be allowed for Part B.

30. Find the area bounded by the spiral $r = \ln\theta$ on the interval $\pi \leqq \theta \leqq 2\pi$.
(A) 2.405 **(B)** 2.931 **(C)** 3.743 **(D)** 4.810 **(E)** 7.487

31. Find the slope of the curve defined parametrically by $x = e^t$, $y = t - \frac{t^3}{9}$ at its smallest x-intercept.
(A) -40.171 **(B)** -2 **(C)** 0.050 **(D)** 0.999 **(E)** 1

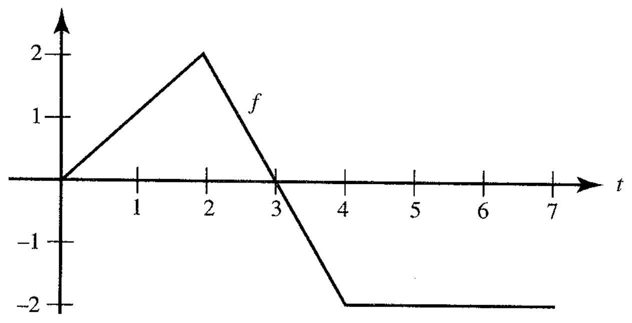
32. $\int \frac{x - 6}{x^2 - 3x} dx =$
(A) $\ln |x^2(x-3)| + C$ **(B)** $-\ln |x^2(x-3)| + C$
(C) $\ln \left|\frac{x^2}{x-3}\right| + C$ **(D)** $\ln \left|\frac{x-3}{x^2}\right| + C$ **(E)** none of these

33. Bacteria in a culture increase at a rate proportional to the number present. An initial population of 200 triples in 10 hours. If this pattern of increase continues unabated, then the approximate number of bacteria after one full day is
(A) 1160 **(B)** 1440 **(C)** 2408 **(D)** 2793 **(E)** 8380

34. Using the substitution $x = 2t - 1$, the definite integral $\int_3^5 t\sqrt{2t-1}\,dt$ may be expressed in the form $k\int_a^b (x+1)\sqrt{x}\,dx$, where $\{k, a, b\} =$
(A) $\left\{\frac{1}{4}, 2, 3\right\}$ **(B)** $\left\{\frac{1}{4}, 3, 5\right\}$ **(C)** $\left\{\frac{1}{4}, 5, 9\right\}$ **(D)** $\left\{\frac{1}{2}, 2, 3\right\}$
(E) $\left\{\frac{1}{2}, 5, 9\right\}$

35. The curve defined by $x^3 + xy - y^2 = 10$ has a vertical tangent line when $x =$
(A) 0 or $-\frac{1}{3}$ **(B)** 1.037 **(C)** 2.074 **(D)** 2.096 **(E)** 2.154

The graph of f shown on [0,7] is for Questions 36 and 37. Let $G(x) = \int_2^{3x-1} f(t)\,dt$.



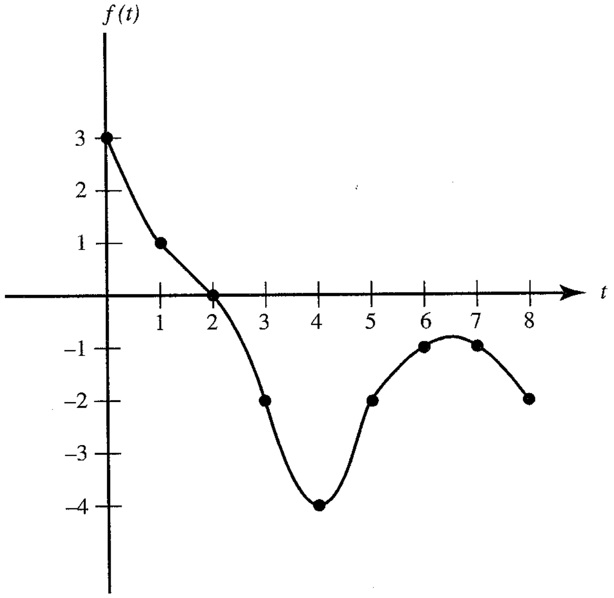
36. $G'(1)$ is
(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** undefined

37. G has a local maximum at $x =$

(A) 1 **(B)** $\frac{4}{3}$ **(C)** 2 **(D)** 5 **(E)** 8

38. If the half-life of a radioactive substance is 8 years, how long will it take, in years, for two thirds of the substance to decay?

(A) 4.68 **(B)** 7.69 **(C)** 12 **(D)** 12.21 **(E)** 12.68



39. Using the left rectangular method, and 4 subintervals, estimate $\int_0^8 |f(t)|\,dt$, where f is the function graphed above.

(A) 4 **(B)** 5 **(C)** 8 **(D)** 15 **(E)** 16

40. The area in the first quadrant bounded by the curve with parametric equations $x = 2a \tan \theta$, $y = 2a \cos^2 \theta$, and the lines $x = 0$ and $x = 2a$ is equal to

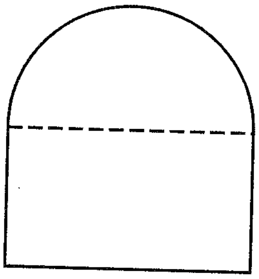
(A) πa^2 **(B)** $2\pi a^2$ **(C)** $\frac{\pi a}{4}$ **(D)** $\frac{\pi a}{2}$ **(E)** none of these

41. The base of a solid is the region bounded by $x^2 = 4y$ and the line $y = 2$, and each plane section perpendicular to the y-axis is a square. The volume of the solid is

(A) 8 **(B)** 16 **(C)** 20 **(D)** 32 **(E)** 64

42. An object initially at rest at (3,3) moves with acceleration $a(t) = (2, e^{-t})$. Where is the object at $t = 2$?

(A) $(4, e^{-2})$ **(B)** $(4, e^{-2} + 2)$ **(C)** $(7, e^{-2})$ **(D)** $(7, e^{-2} + 2)$
(E) $(7, e^{-2} + 4)$



43. The figure shown consists of a rectangle capped by a semicircle. Its area is 100 yd^2. The minimum perimeter of the figure is
(A) 10.584 yd **(B)** 28.284 yd **(C)** 37.793 yd **(D)** 38.721 yd
(E) 51.820 yd

44. Using the first two terms in the Maclaurin series for $y = \cos x$, we can achieve accuracy to within 0.001 over the interval $|x| < k$ when $k =$
(A) 0.032 **(B)** 0.394 **(C)** 0.786 **(D)** 0.788 **(E)** 1.570

45. After t years, $50e^{-0.015t}$ pounds of a deposit of a radioactive substance remains. The average amount per year *not* lost by radioactive decay during the second hundred years is
(A) 2.9 lb **(B)** 5.8 lb **(C)** 7.4 lb **(D)** 11.1 lb **(E)** none of these

Answers to BC Practice Examination 3: Section I

1.	B	**10.**	E	**19.**	E	**28.**	D	**37.**	B
2.	C	**11.**	D	**20.**	C	**29.**	B	**38.**	E
3.	C	**12.**	A	**21.**	B	**30.**	C	**39.**	E
4.	D	**13.**	A	**22.**	D	**31.**	A	**40.**	A
5.	B	**14.**	D	**23.**	D	**32.**	C	**41.**	D
6.	C	**15.**	B	**24.**	B	**33.**	D	**42.**	E
7.	B	**16.**	E	**25.**	B	**34.**	C	**43.**	C
8.	B	**17.**	C	**26.**	A	**35.**	C	**44.**	B
9.	B	**18.**	B	**27.**	E	**36.**	D	**45.**	B

The explanations for questions not given below will be found in the answers to AB Practice Examination 3, on pages 506 to 514. Identical questions in Section I of Practice Examinations AB3 and BC3 have the same number. The answer to Question 4, for example, not given below, will be found in Section I of Examination AB3, Answer 4, page 506.

Part A

1. B. Since $\lim_{x \to 1} f(x) = 1$, to render $f(x)$ continuous at $x = 1$ we must define $f(1)$ to be 1.

2. C. Note that

$$\frac{\sin^2 \frac{x}{2}}{x^2} = \frac{\sin^2 \frac{x}{2}}{4\frac{x^2}{4}} = \frac{1}{4} \lim_{\theta \to 0} \left[\frac{\sin \theta}{\theta}\right]^2,$$

where we let $\frac{x}{2} = \theta$.

3. C. We obtain the first few terms of the Maclaurin series generated by $f(x) = \sqrt{1 + x}$:

$$f(x) = \sqrt{1 + x}; \qquad f(0) = 1;$$

$$f'(x) = \frac{1}{2}(1 + x)^{-1/2}; \qquad f'(0) = \frac{1}{2};$$

$$f''(x) = -\frac{1}{4}(1 + x)^{-3/2}; \qquad f''(0) = -\frac{1}{4};$$

$$f'''(x) = \frac{3}{8}(1 + x)^{-5/2}. \qquad f'''(0) = \frac{3}{8}.$$

So $\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{1}{4} \cdot \frac{x^2}{2} + \frac{3}{8} \cdot \frac{x^3}{6} - \cdots$.

6. C. Here,

$$\frac{dx}{dt} = e^t(\cos t - \sin t), \quad \frac{dy}{dt} = e^t(\sin t + \cos t),$$

and

$$\frac{d^2x}{dt^2} = -2(\sin t)e^t, \quad \frac{d^2y}{dt^2} = 2(\cos t)e^t;$$

and the magnitude of the acceleration, $|\mathbf{a}|$, is given by

$$|\mathbf{a}| = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2} = 2e^t.$$

7. B. The new series is

$$1 + \frac{x-1}{2} + \frac{(x-1)^2}{3} + \frac{(x-1)^3}{4} + \cdots.$$

Using the Ratio Test, page 346, we find that the series converges when $0 < x < 2$. Be sure to check the endpoints!

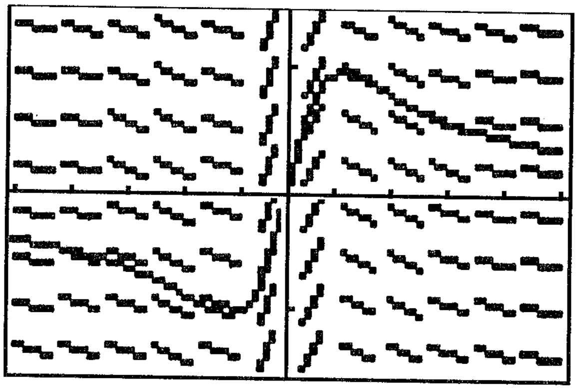
10. E. The integral is equal to $\left.\frac{\tan^3 x}{3}\right|_{\pi/4}^{\pi/3} = \frac{1}{3}(3\sqrt{3} - 1)$.

11. D. $\int_1^e \ln x\, dx$ can be integrated by parts to yield $(x \ln x - x)\Big|_1^e$, which equals

$$e \ln e - e - (1 \ln 1 - 1) = e - e - (0 - 1) = 1.$$

13. A. $\lim\limits_{n\to\infty} \frac{3^n}{n^3} = \infty$; $\frac{3n^2}{n^3+1} > \frac{1}{n}$.

17. C. $\int_0^\infty \frac{dx}{x^2+1} = \lim\limits_{b\to\infty} \tan^{-1} x \Big|_0^b = \frac{\pi}{2}$. The integrals in (A), (B), and (D) all diverge to infinity.



18. B. Solution curves appear to represent odd functions with a horizontal asymptote. We have superimposed the curve in (B) on the slope field.

19. E. At (2,1), $\frac{dy}{dx} = 3$; using $\Delta x = 0.1$, Euler's method moves us to $(2.1, 1 + 3(0.1))$.

At (2.1, 1.3), $\frac{dy}{dx} = 3.4$, so the next point is $(2.2, 1.3 + 3.4(0.1))$.

22. D. Separate variables to get $\frac{dy}{y} = \frac{dx}{x}$, and integrate to get $\ln y = \ln x + C$. Since $y = 3$ when $x = 1$, $C = \ln 3$.

23. D. The generating circle has equation $x^2 + y^2 = 4$. The volume, V, is given by

$$V = \pi \int_{-1}^{1} x^2\, dy = 2\pi \int_0^1 (4 - y^2)\, dy.$$

24. B. The arc length is

$$\int_1^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^4 \sqrt{1+\left(x-\frac{1}{4x}\right)^2}\,dx$$

$$= \int_1^4 \sqrt{\left(x+\frac{1}{4x}\right)^2}\,dx = \int_1^4 \left(x+\frac{1}{4x}\right)dx$$

$$= \frac{15}{2} + \frac{1}{4}\ln 4 = \frac{15}{2} + \frac{1}{2}\ln 2.$$

26. A. Here $x^2 + y^2 = 4$, whose locus is a circle; but since the given equations imply x and y both nonnegative, the curve defined is in the first quadrant.

28. D. $\displaystyle\int_1^2 \frac{1}{\sqrt{4-x^2}}\,dx = \lim_{h\to 2^-} \int_1^h \frac{1}{\sqrt{4-x^2}}\,dx = \lim_{h\to 2^-} \sin^{-1}\frac{x}{2}\Big|_1^h$

$$= \lim_{h\to 2^-}\left(\sin^{-1}\frac{h}{2} - \sin^{-1}\frac{1}{2}\right) = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}.$$

Part B

30. C. Since the equation of the spiral is $r = \ln \theta$, we will use the polar mode. The formula for area in polar coordinates is

$$\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\, d\theta$$

Therefore, calculate

```
.5fnInt((ln θ)²,θ,π,2π)
```

The result is 3.743.

31. A. The curve is given parametrically by

$$x = e^t \quad y = t - \frac{t^3}{9}$$

Since the x-intercepts occur where $y = 0$, we solve the equation $t - \frac{t^3}{9} = 0$, getting $t = -3$, 0, and 3. The smallest x-intercept is for $t = -3$. At that point, the slope

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1-\frac{t^2}{3}}{e^t}$$

At $t = -3$, $\dfrac{dy}{dx} = \dfrac{-2}{e^{-3}} = -40.171.$

32. C. We use the method of partial fractions, letting

$$\frac{x-6}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$$

Then $A = 2$ and $B = -1$.

38. E. If Q_0 is the initial amount of the substance and Q is the amount at time t, then

$$Q = Q_0 e^{-kt}.$$

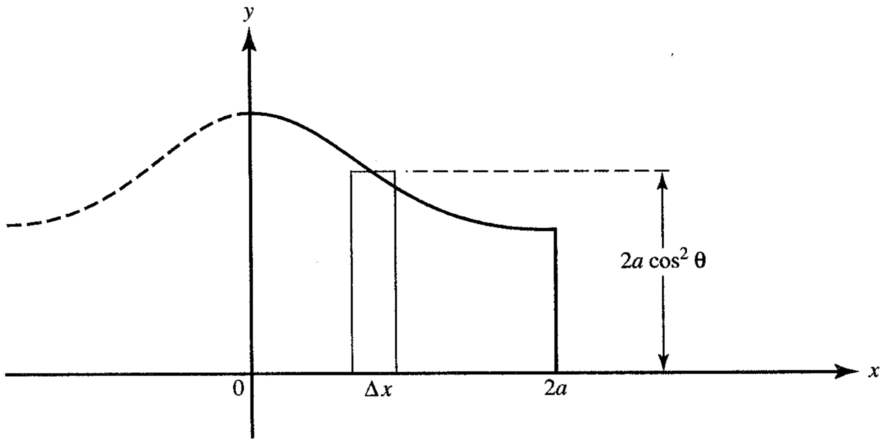
When $t = 8$, $Q = \frac{1}{2} Q_0$, so

$$\frac{1}{2} Q_0 = Q_0 e^{-8k}$$

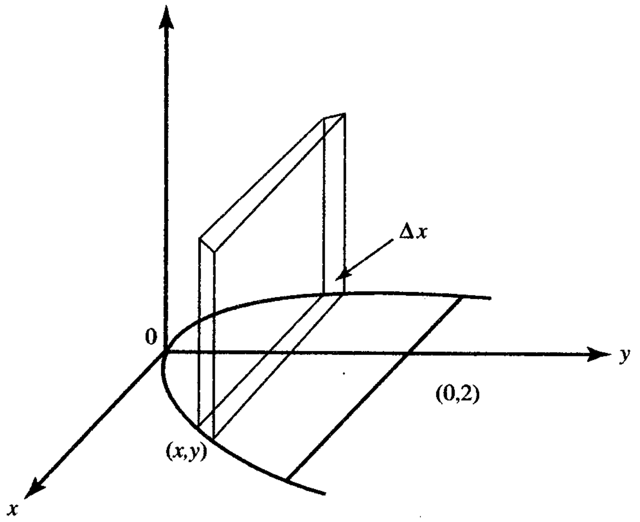
and $\frac{1}{2} = e^{-8k}$. Thus $k = 0.08664$. Using a calculator, we seek t when $Q = \frac{1}{3} Q_0$. $\frac{1}{3} = e^{-0.08664t}$, so $t \approx 12.68$. Don't round off k too quickly.

40. A. (See figure below.)

$$A = \int_0^{2a} y\,dx = \int_{\theta=0}^{\theta=\frac{\pi}{4}} 2a\cos^2\theta \cdot 2a\sec^2\theta\,d\theta = 4a^2\,\theta\Big|_0^{\pi/4} = \pi a^2$$



41. D.



$$\Delta V = (2x)^2\,\Delta y = 4x^2\,\Delta y$$
$$= 16y\,\Delta y$$
$$V = \int_0^2 16y\,dy$$

42. E. $\mathbf{v}(t) = (2t + c_1, -e^{-t} + c_2)$; $\mathbf{v}(0) = (0,0)$ yields $c_1 = 0$ and $c_2 = 1$.
$\mathbf{R}(t) = (t^2 + c_3, e^{-t} + t + c_4)$; $\mathbf{R}(0) = (3,3)$ yields $\mathbf{R}(t) = (t^2 + 3, e^{-t} + t + 2)$.

44. B. We seek k such that $\cos x$ will differ from $\left(1 - \frac{x^2}{2}\right)$ by less than 0.001 at $x = k$. So we evaluate

```
solve(cos X-(1-X²/2)-.001,X,.5)
```

which rounds down to 0.394.