BC Practice Examination 2

Section I

Part A†

(See instructions, page 532. Answers are given on page 551.)

1. $\lim_{x\to\infty} \frac{20x^2 - 13x + 5}{5 - 4x^3}$ is

   (A) -5 (B) ∞ (C) 0 (D) 5 (E) 1

2. $\lim_{h\to 0} \frac{\ln(2+h) - \ln 2}{h}$ is

   (A) 0 (B) $\ln 2$ (C) $\frac{1}{2}$ (D) $\frac{1}{\ln 2}$ (E) ∞

3. If $x = \sqrt{1 - t^2}$ and $y = \sin^{-1} t$, then $\frac{dy}{dx}$ equals

   (A) $-\frac{\sqrt{1-t^2}}{t}$ (B) $-t$ (C) $\frac{t}{1-t^2}$ (D) 2 (E) $-\frac{1}{t}$

The table shown is for Questions 4 and 5. The differentiable functions f and g have the values shown.

x	f	f'	g	g'
1	2	$\frac{1}{2}$	-3	5
2	3	1	0	4
3	4	2	2	3
4	6	4	3	$\frac{1}{2}$

†Beginning in May 1998, 55 minutes will be allowed for Part A.

4. The average rate of change of function f on [1, 4] is
(A) 7/6 **(B)** 4/3 **(C)** 15/8 **(D)** 9/4 **(E)** 8/3

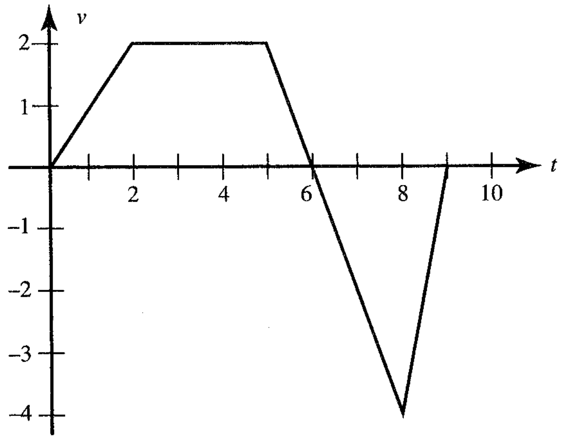
5. If $h(x) = g(f(x))$ then $h'(3) =$
(A) 1/2 **(B)** 1 **(C)** 4 **(D)** 6 **(E)** 9

6. Which one of the following series converges?
(A) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ **(B)** $\sum_{n=1}^{\infty} \frac{1}{n}$ **(C)** $\sum_{n=1}^{\infty} \frac{1}{2n+1}$
(D) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ **(E)** $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

7. The maximum value of the function $f(x) = xe^{-x}$ is
(A) $\frac{1}{e}$ **(B)** e **(C)** 1 **(D)** -1 **(E)** none of these

8. A rectangle of perimeter 18 in. is rotated about one of its sides to generate a right circular cylinder. The rectangle which generates the cylinder of largest volume has area, in square inches, of
(A) 14 **(B)** 20 **(C)** $\frac{81}{4}$ **(D)** 18 **(E)** $\frac{77}{4}$

9. $\int_1^2 (3x - 2)^3\, dx$ is equal to
(A) $\frac{16}{3}$ **(B)** $\frac{63}{4}$ **(C)** $\frac{13}{3}$ (D) $\frac{85}{4}$ **(E)** none of these



For Questions 10 and 11, the graph shows the velocity of an object moving along a line, for $0 \leq t \leq 9$.

10. At what time does the object attain its maximum acceleration?
(A) $2 < t < 5$ **(B)** $5 < t < 8$ **(C)** $t = 6$ **(D)** $t = 8$ **(E)** $8 < t < 9$

11. The object is farthest from the starting point at $t =$
(A) 2 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 9

12. If we let $x = 2\sin\theta$, then $\int_0^2 \frac{x^2 dx}{\sqrt{4 - x^2}}$ is equivalent to:

(A) $4\int_0^1 \sin^2\theta\, d\theta$ **(B)** $\int_0^{\pi/2} 4\sin^2\theta\, d\theta$ **(C)** $\int_0^{\pi/2} 2\sin\theta\tan\theta\, d\theta$

(D) $\int_0^2 \frac{2\sin^2\theta}{\cos^2\theta}\, d\theta$ **(E)** $4\int_{\pi/2}^0 \sin^2\theta\, d\theta$

13. $\int_{-1}^1 (1 - |x|)\, dx$ equals

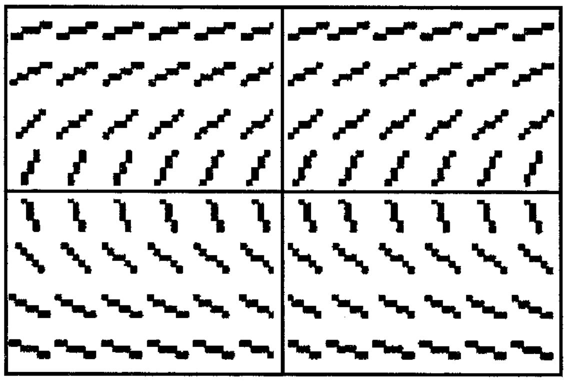
(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** none of these

14. $\lim_{x\to\infty} x^{1/x}$

(A) $= 0$ **(B)** $= 1$ **(C)** $= e$ **(D)** $= \infty$ **(E)** does not exist

15. A population of rabbits grows according to the differential equation $\frac{dR}{dt} = 0.001R(200 - R)$, where $R(t)$ is the number of rabbits after t months. If there were initially 25 rabbits, approximately how many months will it take the population to double?

(A) 2.7 **(B)** 3.5 **(C)** 4.2 **(D)** 6.9 **(E)** 8.4



16. Which equation has the slope field shown above?

(A) $\frac{dy}{dx} = \frac{5}{y}$ **(B)** $\frac{dy}{dx} = \frac{5}{x}$ **(C)** $\frac{dy}{dx} = \frac{x}{y}$ **(D)** $\frac{dy}{dx} = 5y$

(E) $\frac{dy}{dx} = x + y$

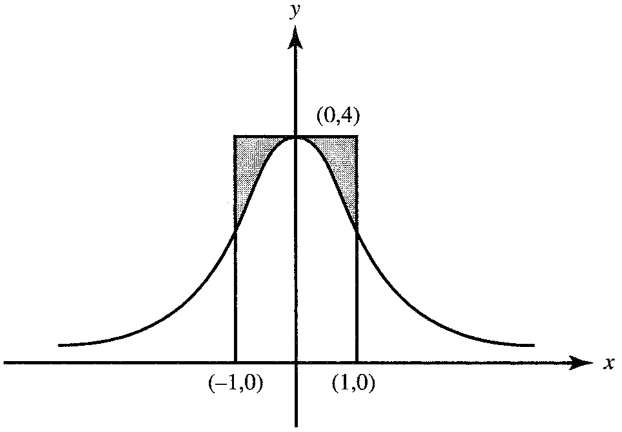
17. Find the slope of the curve $r = \cos 2\theta$ at $\theta = \frac{\pi}{6}$.

(A) $\frac{\sqrt{3}}{7}$ **(B)** $\frac{1}{\sqrt{3}}$ **(C)** 0 **(D)** $\sqrt{3}$ **(E)** $-\sqrt{3}$

18. $\int_0^6 f(x-1)\,dx =$

(A) $\int_{-1}^{7} f(x)\,dx$ **(B)** $\int_{-1}^{5} f(x)\,dx$ **(C)** $\int_{-1}^{5} f(x+1)\,dx$

(D) $\int_{1}^{5} f(x)\,dx$ **(E)** $\int_{1}^{7} f(x)\,dx.$



19. The equation of the curve shown above is $y = \frac{4}{1+x^2}$. What does the area of the shaded region equal?

(A) $4 - \frac{\pi}{4}$ **(B)** $8 - 2\pi$ **(C)** $8 - \pi$ **(D)** $8 - \frac{\pi}{2}$ **(E)** $2\pi - 4$

20. The length of one arch of the cycloid $x = \theta - \sin\theta$, $y = 1 - \cos\theta$ is given by the integral

(A) $\int_0^{\pi} \sin\frac{\theta}{2}\,d\theta$ **(B)** $2\int_0^{\pi} \sin\frac{\theta}{2}\,d\theta$ **(C)** $2\int_0^{2\pi} \sin\frac{\theta}{2}\,d\theta$

(D) $\sqrt{2}\int_0^{\pi} \sqrt{1-\cos\theta}\,d\theta$ **(E)** none of these

21. A particle moves along a line with velocity, in feet per second, $v = t^2 - t$. The total distance, in feet, traveled from $t = 0$ to $t = 2$ equals

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$ **(C)** 2 **(D)** 1 **(E)** $\frac{4}{3}$

22. The general solution of the differential equation $\frac{dy}{dx} = \frac{1-2x}{y}$ is a family of

(A) straight lines **(B)** circles **(C)** hyperbolas **(D)** parabolas
(E) ellipses

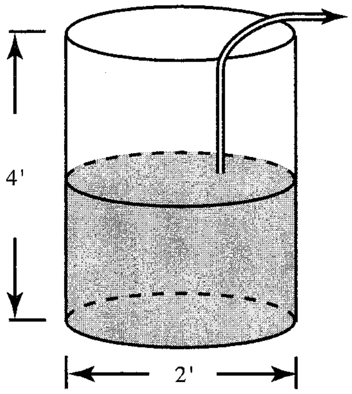
23. The curve $x^3 + x\tan y = 27$ passes through (3, 0). Use local linearization to estimate the value of y at $x = 3.1$. The value is

(A) -2.7 **(B)** -0.9 **(C)** 0 **(D)** 0.1 **(E)** 3.0

24. $\int x \cos x \, dx =$

(A) $x \sin x + \cos x + C$ **(B)** $x \sin x - \cos x + C$

(C) $\frac{x^2}{2} \sin x + C$ **(D)** $\frac{1}{2} \sin x^2 + C$ **(E)** none of these



25. The work done in lifting an object is the product of the weight of the object and the distance it is moved. A cylindrical barrel 2 ft in diameter and 4 ft high is half-full of oil weighing 50 lb per ft^3. How much work is done, in ft-lb, in pumping the oil to the top of the tank?

(A) 100π **(B)** 200π **(C)** 300π **(D)** 400π **(E)** 1200π

26. The coefficient of the x^2-term in the Taylor Polynomial for $y = x^{2/3}$ around $x = 8$ is

(A) $-\frac{1}{144}$ **(B)** $-\frac{1}{72}$ **(C)** $-\frac{1}{9}$ **(D)** $\frac{1}{144}$ **(E)** $\frac{1}{6}$

27. If $f'(x) = h(x)$ and $g(x) = x^3$, then $\frac{d}{dx} f(g(x)) =$

(A) $h(x^3)$ **(B)** $3x^2h(x)$ **(C)** $h'(x)$ **(D)** $3x^2h(x^3)$ **(E)** $x^3h(x^3)$

28. $\int_0^\infty e^{-x/2} \, dx =$

(A) $-\infty$ **(B)** -2 **(C)** 1 **(D)** 2 **(E)** ∞

Part B†

(See instructions, page 536.)

29. The path of a satellite is given by the parametric equations

$$x = 4 \cos t + \cos 12t$$

$$y = 4 \sin t + \sin 12t.$$

The upward velocity at $t = 1$ equals

(A) 2.829 **(B)** 3.005 **(C)** 3.073 **(D)** 3.999 **(E)** 12.287

†Beginning in May 1998, 50 minutes will be allowed for Part B.

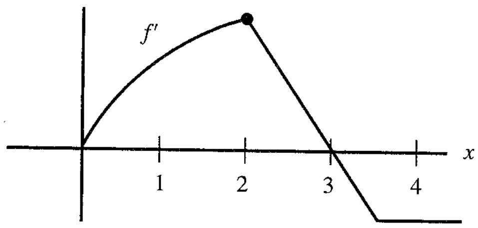
30. A differentiable function has values shown in this table:
Estimate $f'(2.1)$.

(A) 0.34 **(B)** 0.59 **(C)** 1.56 **(D)** 1.70 **(E)** 1.91

x	2.0	2.2	2.4	2.6	2.8	3.0
$f(x)$	1.39	1.73	2.10	2.48	2.88	3.30

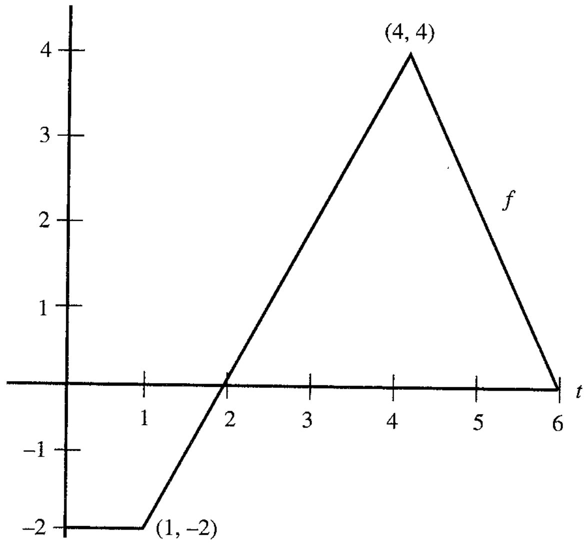
31. An object moving along a line has velocity $v(t) = t \cos t - \ln(t + 2)$, where $0 \le t \le 10$. The object achieves its maximum speed when $t =$

(A) 3.743 **(B)** 5.107 **(C)** 6.419 **(D)** 7.550 **(E)** 9.538



32. The graph of f' shown above consists of a quarter-circle and two line-segments. At $x = 2$ which of the following statements is true?

(A) f is not continuous.
(B) f is continuous but not differentiable.
(C) f has a relative maximum.
(D) f has a point of inflection.
(E) none of these.



33. Let $H(x) = \int_0^x f(t)\,dt$, where f is the function whose graph appears above:

The local linearization of $H(x)$ near $x = 3$ is $H(x) \simeq$

(A) $-2x + 8$ **(B)** $2x - 4$ **(C)** $-2x + 4$ **(D)** $2x - 8$ **(E)** $2x - 2$

34. The table shows the speed of an object in feet per second during a three-second period. Estimate the distance the object travels, using the trapezoid method:
(A) 34 ft **(B)** 45 ft **(C)** 48 ft **(D)** 49 ft **(E)** 64 ft

time (sec)	0	1	2	3
speed (ft/sec)	30	22	12	0

35. In a marathon, when the winner crosses the finish line many runners are still on the course, some quite far behind. If the density of runners x miles from the finish line is given by $R(x) = 20[1 - \cos(1 + .03x^2)]$ runners per mile, how many are within 8 miles of the finish line?
(A) 30 **(B)** 145 **(C)** 157 **(D)** 166 **(E)** 195

36. Find the volume of the solid generated when the region bounded by the y-axis, $y = e^x$, and $y = 2$ is rotated around the y-axis.
(A) 0.296 **(B)** 0.592 **(C)** 2.427 **(D)** 3.998 **(E)** 27.577

37. If $f(t) = \int_0^{t^2} \frac{1}{1 + x^2}\,dx$, then $f'(t)$ equals

(A) $\frac{1}{1 + t^2}$ **(B)** $\frac{2t}{1 + t^2}$ **(C)** $\frac{1}{1 + t^4}$ **(D)** $\frac{2t}{1 + t^4}$ **(E)** $\tan^{-1} t^2$

38. We wish to estimate e^x, over the interval $|x| < 2$, with an error less than 0.001. The Lagrange error term suggests that we use a Taylor polynomial at 0 with degree at least
(A) 6 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

39. Find the volume of the solid formed when one arch of the cycloid defined parametrically by $x = \theta - \sin\theta$, $y = 1 - \cos\theta$ is rotated around the x-axis.
(A) 15.708 **(B)** 17.306 **(C)** 19.739 **(D)** 29.609 **(E)** 49.348

40. If $y = \frac{x - 3}{2 - 5x}$, then $\frac{dy}{dx}$ equals
(A) $\frac{17 - 10x}{(2 - 5x)^2}$ **(B)** $\frac{13}{(2 - 5x)^2}$ **(C)** $\frac{x - 3}{(2 - 5x)^2}$ **(D)** $\frac{17}{(2 - 5x)^2}$
(E) $\frac{-13}{(2 - 5x)^2}$

41. For which function is $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ the Taylor series about 0?
(A) e^x **(B)** e^{-x} **(C)** $\sin x$ **(D)** $\cos x$ **(E)** $\ln(1 + x)$

42. The hypotenuse AB of a right triangle ABC is 5 ft, and one leg, AC, is decreasing at the rate of 2 ft/sec. The rate, in square feet per second, at which the area is changing when $AC = 3$ is
(A) $\frac{25}{4}$ **(B)** $\frac{7}{4}$ **(C)** $-\frac{3}{2}$ **(D)** $-\frac{7}{4}$ **(E)** $-\frac{7}{2}$

43. If $f'(x)$ exists on the closed interval $[a, b]$, then it follows that
(A) $f(x)$ is constant on $[a, b]$
(B) there exists a number c, $a < c < b$, such that $f'(c) = 0$
(C) the function has a maximum value on the open interval (a, b)
(D) the function has a minimum value on the open interval (a, b)
(E) the Mean Value Theorem applies

44. If $\frac{dy}{dx} = y \tan x$ and $y = 3$ when $x = 0$, then, when $x = \frac{\pi}{3}$, $y =$

(A) $\ln \sqrt{3}$ **(B)** $\ln 3$ **(C)** $\frac{3}{2}$ **(D)** $\frac{3\sqrt{3}}{2}$ **(E)** 6

45. The rate at which a purification process can remove contaminants from a tank of water is proportional to the amount of contaminant remaining. If 20% of the contaminant can be removed during the first minute of the process and 98% must be removed to make the water safe, approximately how long will the decontamination process take?
(A) 2 min **(B)** 5 min **(C)** 18 min **(D)** 20 min **(E)** 40 min

Answers to BC Practice Examination 2: Section I

1.	C	**10.**	E	**19.**	B	**28.**	D	**37.**	D
2.	C	**11.**	C	**20.**	C	**29.**	E	**38.**	C
3.	E	**12.**	B	**21.**	D	**30.**	D	**39.**	E
4.	B	**13.**	C	**22.**	E	**31.**	E	**40.**	E
5.	B	**14.**	B	**23.**	B	**32.**	D	**41.**	D
6.	E	**15.**	C	**24.**	A	**33.**	D	**42.**	D
7.	A	**16.**	A	**25.**	C	**34.**	D	**43.**	E
8.	D	**17.**	A	**26.**	A	**35.**	D	**44.**	E
9.	D	**18.**	B	**27.**	D	**36.**	B	**45.**	C

The explanation for questions not given below will be found in the answers to AB Practice Examination 2, on pages 448 to 452. Identical questions in Section I of Practice Examinations AB2 and BC2 have the same number. The answer to Question 1, for example, not given below, will be found in Section I of Examination AB2, answer 1, page 491.

Part A

3. E. Here,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{\sqrt{1-t^2}}}{\frac{1}{2}\frac{(-2t)}{\sqrt{1-t^2}}} = -\frac{1}{t}.$$

6. E. Each is essentially a p-series, $\sum \frac{1}{n^p}$. Such a series converges only if $p > 1$.

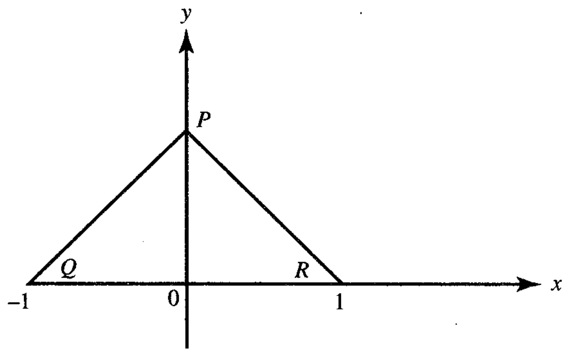
7. A. Here, $f'(x)$ is $e^{-x}(1-x)$; f has maximum value when $x = 1$.

9. D. Evaluate $\left.\frac{1}{12}(3x-2)^4\right|_1^2$.

12. B. Note that, when $x = 2\sin\theta$, $x^2 = 4\sin^2\theta$, $dx = 2\cos\theta\, d\theta$, and $\sqrt{4-x^2} = 2\cos\theta$. Also,

$$\text{when } x = 0,\ \theta = 0;$$

$$\text{when } x = 2,\ \theta = \frac{\pi}{2}.$$



13. C. The given integral is equivalent to $\int_{-1}^{0}(1+x)\,dx + \int_{0}^{1}(1-x)\,dx$.

The figure shows the graph of $f(x) = 1 - |x|$ on $[-1,1]$.

The area of triangle PQR is equal to $\int_{-1}^{1}(1-|x|)\,dx$.

14. B. Let $y = x^{1/x}$; then take logarithms. $\ln y = \frac{\ln x}{x}$. As $x \to \infty$, the fraction is of the form ∞/∞. $\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{1/x}{1} = 0$. So $y \to e^0$ or 1.

15. C. We solve using separation of variables and partial fractions.

$$\frac{dR}{R(200-R)} = 0.001\,dt$$

$$\frac{1}{200}\int\left(\frac{1}{R} + \frac{1}{200-R}\right)dR = 0.001\,dt$$

$$\frac{1}{200}\ln\left(\frac{R}{200-R}\right) = 0.001t + C$$

$$\frac{200-R}{R} = ce^{-0.2t} \quad \rightarrow \quad R = \frac{200}{1+ce^{-0.2t}}$$

Initial condition $R = 25$ at $t = 0$ yields $c = 7$. Now solve

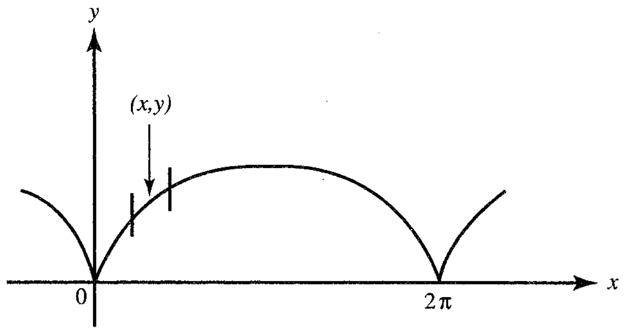
$$50 = \frac{200}{1+7e^{-0.2t}} \text{ to find } t \approx 4.236.$$

16. A. Note that (1) on a horizontal line the slope segments are all parallel; so the slopes there are all the same and $\frac{dy}{dx}$ must depend only on y; (2) along the x-axis (where $y = 0$) the slopes are infinite, and (3) as y increases, the slope decreases.

17. A. We can represent the coordinates parametrically as $(r\cos\theta, r\sin\theta)$. Now

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\cos\theta + \frac{dr}{d\theta}\cdot\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cdot\cos\theta}$$

Note that $\frac{dr}{d\theta} = -2\sin 2\theta$. Then evaluate $\frac{dy}{dx}$ at $\theta = \frac{\pi}{6}$. (Alternatively, we can write $x = \cos 2\theta\cos\theta$ and $y = \cos 2\theta\sin\theta$ to find $\frac{dy}{dx}$ from $\frac{dy/d\theta}{dx/d\theta}$.)



20. C. $\dfrac{ds}{d\theta} = \sqrt{\left(\dfrac{dx}{d\theta}\right)^2 + \left(\dfrac{dy}{d\theta}\right)^2} = \sqrt{(1 - \cos\theta)^2 + (\sin\theta)^2}$

$$= \sqrt{2(1 - \cos\theta)} = \sqrt{2 \cdot 2\sin^2\frac{\theta}{2}} = 2\sin\frac{\theta}{2}.$$

So each arch has length $2\int_0^{2\pi} \sin\frac{\theta}{2}\, d\theta$.

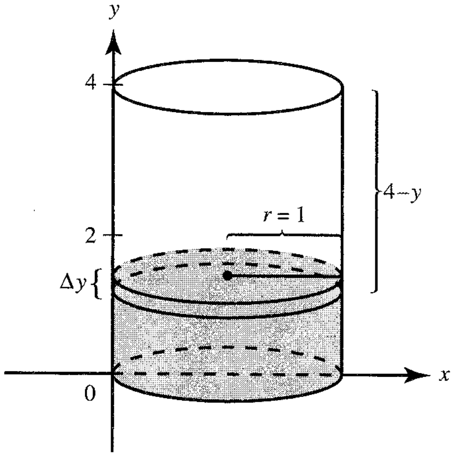
21. D. Note that v is negative from $t = 0$ to $t = 1$, but positive from $t = 1$ to $t = 2$. Thus the distance traveled is given by

$$-\int_0^1 (t^2 - t)\, dt + \int_1^2 (t^2 - t)\, dt.$$

22. E. Separating variables, we get $y\, dy = (1 - 2x)\, dx$. Integrating gives

$$\frac{1}{2}y^2 = x - x^2 + C \text{ or } y^2 = 2x - 2x^2 + k \text{ or } 2x^2 + y^2 - 2x = k.$$

24. A. Using parts: $u = x$, $dv = \cos x\, dx$; $du = dx$, $v = \sin x$; thus,
$\int x\cos x\, dx = x\sin x - \int \sin x\, dx.$



25. C. Consider a thin slice of the oil, and the work Δw done in raising it to the top of the tank:

$$\Delta w = (\text{weight of oil}) \times (\text{distance raised})$$
$$= (50 \cdot \pi \cdot 1^2 \,\Delta y)(4 - y)$$

The total work is thus $50\pi \int_0^2 (4 - y)\, dy$

26. A. The Taylor polynomial of degree 2 at $x = 8$ for $f(x) = x^{2/3}$ is

$$x^{2/3} = 4 + \frac{1}{3}(x - 8) - \frac{1}{72 \cdot 2!}(x - 8)^2$$

27. D. Here,

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x) = h(g(x))g'(x) = h(x^3) \cdot 3x^2.$$

28. D. Evaluate

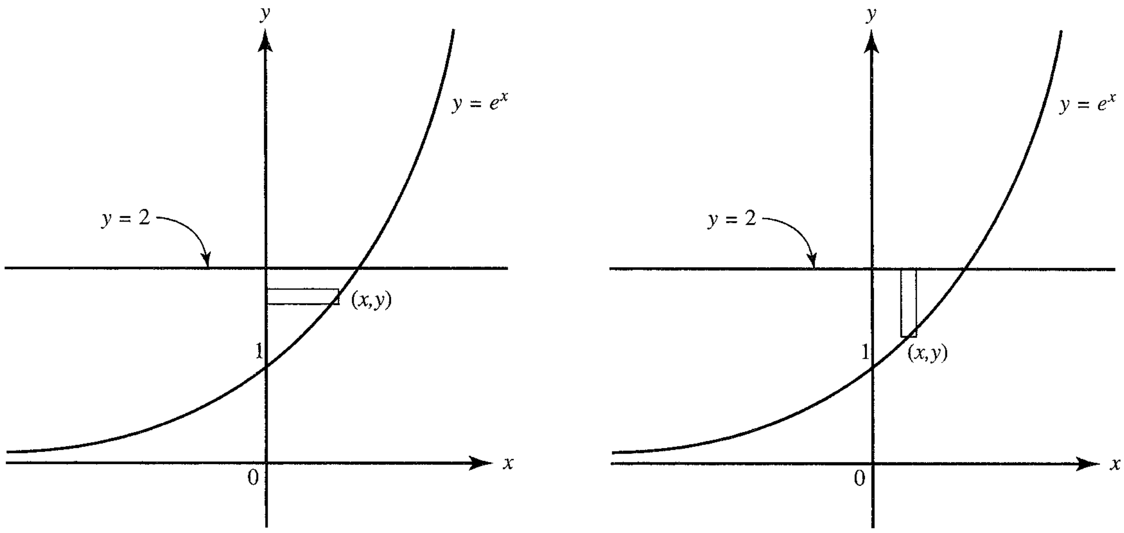
$$\lim_{b \to \infty} \int_0^b e^{-x/2}\, dx = -\lim_{b \to \infty} 2e^{-x/2} \Big|_0^b = -2(0 - 1).$$

Part B

29. E. The vertical component of velocity is

$$\frac{dy}{dt} = 4 \cos t + 12 \cos 12t$$

Evaluate at $t = 1$.



36. B. Using disks, $\Delta V = \pi R^2 H = \pi(\ln y)^2\ \Delta y$. Note that the limits of the definite integral are 1 and 2. With change of variables for the calculator, we evaluate the expression

```
πfnInt((lnX)²,X,1,2)
```

The required volume is 0.592.

38. C. The Maclaurin expansion is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$$

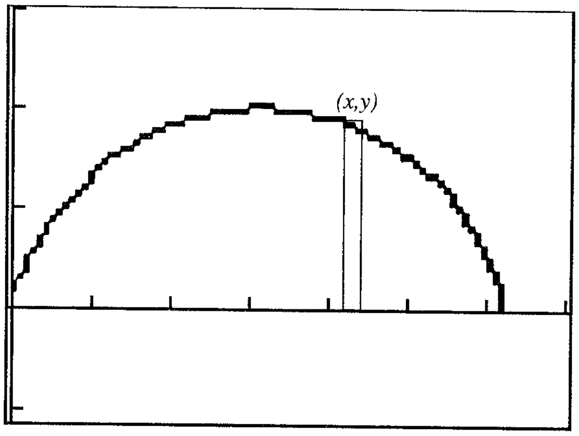
The Lagrange remainder R, after n terms, for some c in the interval $|x| \leqq 2$, is

$$R = \frac{f^{(n+1)}(c) \cdot c^{n+1}}{(n+1)!} = \frac{e^c c^{n+1}}{(n+1)!}$$

Since R is greatest when $c = 2$, n needs to satisfy the inequality

$$\frac{e^2 2^{n+1}}{(n+1)!} < 0.001$$

Set $\mathtt{Y_1}$ equal to `(e²)(2^(X+1))/(X+1)!` Evaluating $\mathtt{Y_1}$ successively at various integral values of `X`, we get $\mathtt{Y_1}(8) > .01$, $\mathtt{Y_1}(9) > .002$, $\mathtt{Y_1}(10) < 3.8 \times 10^{-4} < .0004$. So we achieve the desired accuracy with a Taylor polynomial at 0 of degree at least 10.



39. E. Using the parametric mode, let `X1T=T-sin T` and `Y1T=1-cos T`. Graph one arch of the cycloid by setting `T` in $[0, 2\pi]$ and `(X,Y)` in $[0, 7] \times [-1, 3]$. Using disks, the desired volume is

$$V = \pi \int_{t=0}^{t=2\pi} y^2\, dx$$

$$= \pi \int_{0}^{2\pi} (1 - \cos t)^2 (1 - \cos t)\, dt$$

We evaluate either

`πfnInt((1-cos T)³,T,0,2π)` or `πfnInt(Y1T³,T,0,2π)`

which yields 49.348.

41. D. See series (3) on page 367.

43. E. The Mean Value Theorem holds if (1) $f(x)$ is continuous on $[a, b]$ and (2) $f'(x)$ exists on (a, b). The condition given in the question assures both of these. Find counterexamples for (A) through (D).

44. E. Separating variables yields $\frac{dy}{y} = \tan x$, so $\ln y = -\ln \cos x + C$. With $y = 3$ when $x = 0$, $C = \ln 3$. The general solution is therefore $(\cos x)\, y = 3$. When $x = \frac{\pi}{3}$,

$$\cos x = \frac{1}{2} \quad \text{and} \quad y = 6.$$