# BC Practice Examination 1

## Section I

### Part A†

The use of calculators is not permitted for this part of the examination.
(Answers are given on page 539).

There are 28 questions in Part A. To make up for possible guessing, the grade on this part is determined by subtracting one-fourth of the number of wrong answers from the number answered correctly.

**1.** $\lim\limits_{x\to\infty} \dfrac{3x^2-4}{2-7x-x^2}$ is

**(A)** 3 **(B)** 1 **(C)** $-3$ **(D)** $\infty$ **(E)** 0

**2.** $\lim\limits_{h\to 0} \dfrac{\cos\left(\frac{\pi}{2}+h\right)}{h}$ is

**(A)** 1 **(B)** nonexistent **(C)** 0 **(D)** $-1$ **(E)** none of these

**3.** If, for all $x$, $f'(x) = (x-2)^4(x-1)^3$, it follows that the function $f$ has

**(A)** a relative minimum at $x = 1$
**(B)** a relative maximum at $x = 1$
**(C)** both a relative minimum at $x = 1$ and a relative maximum at $x = 1$
**(D)** neither a relative maximum nor a relative minimum
**(E)** relative minima at $x = 1$ and at $x = 2$

**4.** Let $F(x) = \displaystyle\int_0^x \frac{10}{1+e^t}\,dt$. Which of the following statements are true?

**I.** $F'(0) = 5$ **II.** $F(2) < F(6)$ **III.** $F$ is concave upward

**(A)** I only **(B)** II only **(C)** III only **(D)** I and II **(E)** I and III

**5.** If $f(x) = 10^x$ and $10^{1.04} \simeq 10.96$, which is closest to $f'(1)$?

**(A)** 0.24 **(B)** 0.92 **(C)** 0.96 **(D)** 10.5 **(E)** 24

†Beginning in May 1998, 55 minutes will be allowed for Part A.

6. If $f$ is differentiable, we can use the line tangent to $f$ at $x = a$ to approximate values of $f$ near $x = a$. Suppose this method always underestimates the correct values. If so, then at $x = a$, $f$ must be

**(A)** positive **(B)** increasing **(C)** decreasing
**(D)** concave upward **(E)** concave downward

7. The region in the first quadrant bounded by the $x$-axis, the $y$-axis, and the curve of $y = e^{-x}$ is rotated about the $x$-axis. The volume of the solid obtained is equal to

**(A)** $\pi$ **(B)** $2\pi$ **(C)** $\frac{1}{2}$ **(D)** $\frac{\pi}{2}$ **(E)** none of these

8. $\int_0^1 \frac{x\,dx}{x^2 + 1}$ is equal to

**(A)** $\frac{\pi}{4}$ **(B)** $\ln \sqrt{2}$ **(C)** $\frac{1}{2}(\ln 2 - 1)$ **(D)** $\frac{3}{2}$ **(E)** $\ln 2$

9. $\lim_{x \to 0^+} x^x$

**(A)** $= 0$ **(B)** $= 1$ **(C)** $= e$ **(D)** $= \infty$ **(E)** does not exist

The table shows the values of differentiable functions $f$ and $g$, for Questions 10 and 11.

| $x$ | $f$ | $f'$ | $g$ | $g'$ |
|---|---|---|---|---|
| 1 | 2 | $\frac{1}{2}$ | $-3$ | 5 |
| 2 | 3 | 1 | 0 | 4 |
| 3 | 4 | 2 | 2 | 3 |
| 4 | 6 | 4 | 3 | $\frac{1}{2}$ |

10. If $P(x) = g^2(x)$, then $P'(3)$ equals

**(A)** 4 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 18

11. If $H(x) = f^{-1}(x)$, then $H'(3)$ equals

**(A)** $-\frac{1}{16}$ **(B)** $-\frac{1}{8}$ **(C)** $-\frac{1}{2}$ **(D)** $\frac{1}{2}$ **(E)** 1

12. $\int_0^1 xe^x\,dx$ equals

**(A)** 1 **(B)** $-1$ **(C)** $2 - e$ **(D)** $\frac{e^2}{2} - e$ **(E)** $e - 1$

13. The curve of $y = \frac{1 - x}{x - 3}$ is concave up when

**(A)** $x > 3$ **(B)** $1 < x < 3$ **(C)** $x > 1$ **(D)** $x < 1$ **(E)** $x < 3$

**14.** The area of the largest isosceles triangle that can be drawn with one vertex at the origin and with the others on a line parallel to and above the $x$-axis and on the curve $y = 27 - x^2$ is

**(A)** 108 **(B)** 27 **(C)** $12\sqrt{3}$ **(D)** 3 **(E)** $24\sqrt{3}$

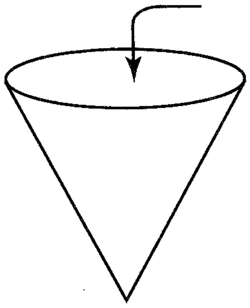
**15.** The length of the curve $y = 2x^{3/2}$ between $x = 0$ and $x = 1$ is equal to

**(A)** $\frac{2}{27}(10^{3/2})$ **(B)** $\frac{2}{27}(10^{3/2} - 1)$ **(C)** $\frac{2}{3}(10^{3/2})$ **(D)** $\frac{4}{5}$

**(E)** none of these

**16.** If $\frac{dx}{dt} = kx$, and if $x = 2$ when $t = 0$ and $x = 6$ when $t = 1$, then $k$ equals

**(A)** ln 4 **(B)** 8 **(C)** $e^3$ **(D)** 3 **(E)** none of these



**17.** Water is poured at a constant rate into a conical reservoir (shown above). If the depth of the water is graphed as a function of time, the graph is

**(A)** decreasing **(B)** constant
**(C)** linear **(D)** concave upward
**(E)** concave downward

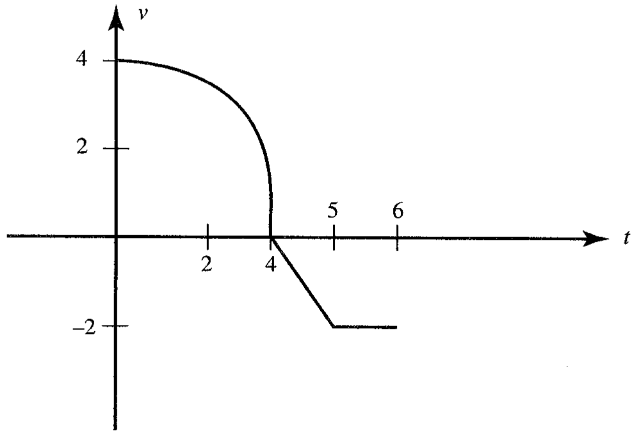
**18.** A particle moves along the curve given parametrically by $x = \tan t$ and $y = \sec t$. At the instant when $t = \frac{\pi}{6}$, its speed equals

**(A)** $\sqrt{2}$ **(B)** $2\sqrt{7}$ **(C)** $\frac{2\sqrt{5}}{3}$ **(D)** $\frac{2\sqrt{13}}{3}$ **(E)** none of these

**19.** Suppose $\frac{dy}{dx} = \frac{10x}{x + y}$ and $y = 2$ when $x = 0$. Using Euler's method with two steps, an estimate of $y$ at $x = 1$ is

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** $5\frac{1}{3}$

The graph shown is for Questions 20 and 21. It consists of a quarter-circle and two line segments, and represents the velocity of an object during the six-second interval.

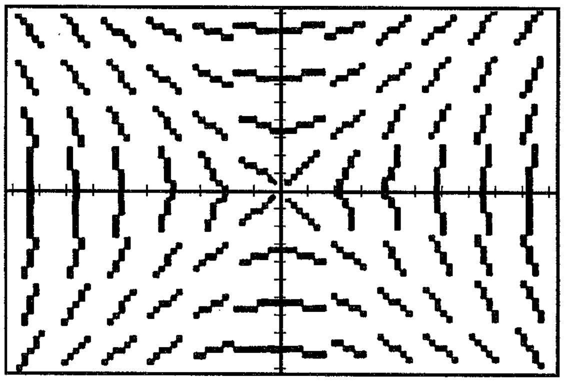



**20.** The object's average speed during the six-second interval is

(A) $\frac{4\pi + 3}{6}$ (B) $\frac{4\pi - 3}{6}$ (C) $-1$ (D) $-\frac{1}{3}$ (E) 1

**21.** The object's acceleration at $t = 2$ is

(A) $-1$ (B) $-\frac{1}{2}$ (C) $-\frac{1}{3}$ (D) $-\frac{1}{\sqrt{3}}$ (E) $-\sqrt{3}$



**22.** Which of the following equations can be a solution of the differential equation whose slope field is shown above?

(A) $2xy = 1$ (B) $2x + y = 1$ (C) $2x^2 + y^2 = 1$ (D) $2x^2 - y^2 = 1$
(E) $y = 2x^2 + 1$

**23.** Which series diverges?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{5^n}$ (B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$ (C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n}}$ (D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n + 1}$

(E) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{5n + 1}$

**24.** If we use the substitution $x = \sin\theta$, which integral is equivalent to $\int_0^1 \frac{\sqrt{1-x^2}}{x}\,dx$?

**(A)** $\int_{\pi/4}^{0} \cot\theta\, d\theta$ **(B)** $\int_0^{\pi/2} \cot\theta\, d\theta$ **(C)** $\int_0^{\pi/2} \frac{\cos^2\theta}{\sin\theta}\, d\theta$

**(D)** $\int_0^1 \frac{\cos^2\theta}{\sin\theta}\, d\theta$ **(E)** none of these

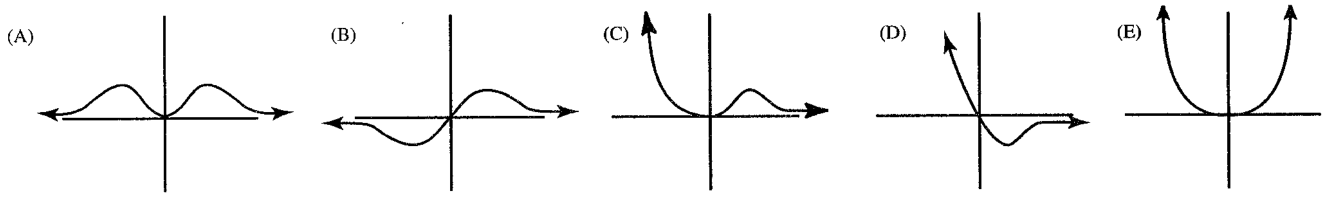
**25.** $\int_a^x g(t)\,dt - \int_b^x g(t)\,dt$ is equal to the constant

**(A)** 0 **(B)** $b - a$ **(C)** $a - b$ **(D)** $\int_a^b g(t)\,dt$ **(E)** $g(b) - g(a)$

**26.** The graph of the pair of parametric equations $x = \sin t - 2$, $y = \cos^2 t$ is part of
**(A)** a circle **(B)** a parabola **(C)** a hyperbola **(D)** a line
**(E)** a cycloid

**27.** The base of a solid is the region bounded by the parabola $y^2 = 4x$ and the line $x = 2$. Each plane section perpendicular to the $x$-axis is a square. The volume of the solid is
**(A)** 6 **(B)** 8 **(C)** 10 **(D)** 16 **(E)** 32

**28.** Which of the following could be the graph of $y = \frac{x^2}{e^x}$?




## Part B†

Some questions in this part of the examination require the use of a graphing calculator. (Answers begin on page 540.)

There are 17 questions in Part B. The penalty for guessing on this part is the same as that for Part A.

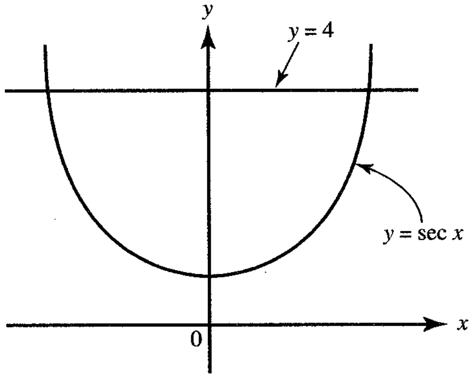
If the exact numerical value of the correct answer is not listed as a choice, select the choice that is closest to the exact numerical answer.

**29.** When partial fractions are used, the decomposition of $\frac{x-1}{x^2+3x+2}$ is equal to

**(A)** $\frac{2}{x+1} - \frac{3}{x+2}$ **(B)** $-\frac{2}{x+1} + \frac{3}{x+2}$ **(C)** $\frac{3}{x+1} - \frac{2}{x+2}$

**(D)** $\frac{2}{x+1} + \frac{3}{x+2}$ **(E)** $-\frac{2}{x+1} - \frac{3}{x+2}$

†Beginning in May 1998, 50 minutes will be allowed for Part B.




**30.** The region $S$ in the figure is bounded by $y = \sec x$ and $y = 4$. What is the volume of the solid formed when $S$ is rotated about the $x$-axis?
**(A)** 0.304 **(B)** 39.867 **(C)** 53.126 **(D)** 54.088 **(E)** 108.177

**31.** The series

$$(x-1) - \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!} - \frac{(x-1)^4}{4!} + \ldots$$

converges
**(A)** for all real $x$ **(B)** if $0 \leqq x < 2$ **(C)** if $0 < x \leqq 2$ **(D)** only if $x = 1$
**(E)** for all $x$ except $0 < x < 2$

**32.** If $f(x)$ is continuous at the point where $x = a$, which of the following statements may be false?
**(A)** $\lim_{x \to a} f(x)$ exists. **(B)** $\lim_{x \to a} f(x) = f(a)$. **(C)** $f'(a)$ exists.
**(D)** $f(a)$ is defined. **(E)** $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$.

**33.** A Maclaurin polynomial is to be used to approximate $y = \sin x$ on the interval $-\pi \leqq x \leqq \pi$. What is the least number of terms needed to guarantee no error greater than 0.1?
**(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** none of these

**34.** Find the area bounded by the $y$-axis and the curve defined parametrically by $x(t) = 4 - t^2$, $y(t) = 2^t$.
**(A)** 6.328 **(B)** 8.916 **(C)** 10.667 **(D)** 12.190 **(E)** 74.529

**35.** If $y$ is a differentiable function of $x$, then the slope of the curve of $xy^2 - 2y + 4y^3 = 6$ at the point where $y = 1$ is
**(A)** $-\frac{1}{18}$ **(B)** $-\frac{1}{26}$ **(C)** $\frac{5}{18}$ **(D)** $-\frac{11}{18}$ **(E)** 2

**36.** If $x = 2t - 1$ and $y = 3 - 4t^2$, then $\frac{dy}{dx}$ is
**(A)** $4t$ **(B)** $-4t$ **(C)** $-\frac{1}{4t}$ **(D)** $2(x + 1)$ **(E)** $-4(x + 1)$

**37.** If $\frac{dy}{dx} = \cos x \cos^2 y$ and $y = \frac{\pi}{4}$ when $x = 0$, then

**(A)** $\tan y = \sin x + 1$ **(B)** $\tan y = -\sin x + 1$ **(C)** $\sec^2 y = \sin x + 2$

**(D)** $\tan y = \frac{1}{2}(\cos^2 x + 1)$ **(E)** $\tan y = \sin x - \frac{\sqrt{2}}{2}$

**38.** If the area under $y = \sin x$ is equal to the area under $y = x^2$ between $x = 0$ and $x = k$, then $k =$

**(A)** $-1.105$ **(B)** 0.877 **(C)** 1.105 **(D)** 1.300 **(E)** 1.571

**39.** The rate at which a rumor spreads across a campus of college students is given by $\frac{dP}{dt} = 0.16(1200 - P)$, where $P(T)$ represents the number of students who have heard the rumor after $t$ days. If 200 students heard the rumor today, how many will have heard it by midnight the day after tomorrow?

**(A)** 320 **(B)** 474 **(C)** 494 **(D)** 520 **(E)** 726

**40.** If $y = x^2 \ln x$ $(x > 0)$, then $y''$ is equal to

**(A)** $3 + \ln x$ **(B)** $3 + 2\ln x$ **(C)** $3\ln x$ **(D)** $3 + 3\ln x$

**(E)** $2 + x + \ln x$

**41.** A 26-ft ladder leans against a building so that its foot moves away from the building at the rate of 3 ft/sec. When the foot of the ladder is 10 ft from the building, the top is moving down at the rate of $r$ ft/sec, where $r$ is

**(A)** $\frac{46}{3}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{5}{4}$ **(D)** $\frac{5}{2}$ **(E)** $\frac{4}{5}$

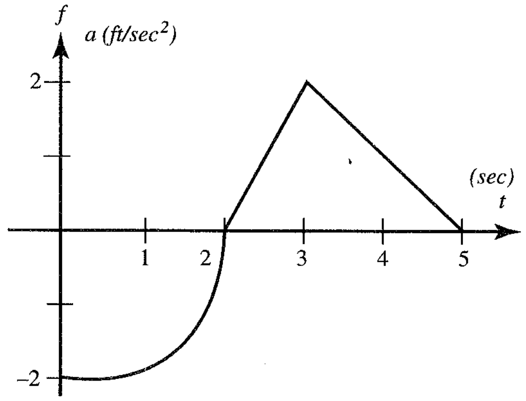
**42.** The coefficient of $x^3$ in the Taylor series of $\ln(1 - x)$ about $x = 0$ (the so-called Maclaurin series) is

**(A)** $-\frac{2}{3}$ **(B)** $-\frac{1}{2}$ **(C)** $-\frac{1}{3}$ **(D)** 0 **(E)** $\frac{1}{3}$

**43.** The graph shows an object's acceleration (in ft/sec$^2$). It consists of a quarter-circle and two line segments. If the object was at rest at $t = 5$ sec, what was its initial velocity?

**(A)** $-2$ ft/sec **(B)** $3 - \pi$ ft/sec **(C)** 0 ft/sec **(D)** $\pi - 3$ ft/sec

**(E)** $\pi + 3$ ft/sec



**44.** Water is leaking from a tank at the rate of $R(t) = 5 \arctan\left(\frac{t}{5}\right)$ gallons per hour, where $t$ is the number of hours since the leak began. How many gallons will leak out during the first day?

**(A)** 7 **(B)** 82 **(C)** 124 **(D)** 141 **(E)** 164

**45.** Find the first-quandrant area inside the rose $r = 3 \sin 2\theta$ but outside the circle $r = 2$.
**(A)** 0.393 **(B)** 0.554 **(C)** 0.790 **(D)** 1.328 **(E)** 2.657

# Answers to BC Practice Examination 1: Section I

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | C | **10.** | D | **19.** | C | **28.** | C | **37.** | A |
| **2.** | D | **11.** | E | **20.** | A | **29.** | B | **38.** | D |
| **3.** | A | **12.** | A | **21.** | D | **30.** | E | **39.** | B |
| **4.** | D | **13.** | E | **22.** | D | **31.** | A | **40.** | B |
| **5.** | E | **14.** | D | **23.** | E | **32.** | C | **41.** | C |
| **6.** | D | **15.** | B | **24.** | C | **33.** | B | **42.** | C |
| **7.** | D | **16.** | E | **25.** | D | **34.** | B | **43.** | D |
| **8.** | B | **17.** | E | **26.** | B | **35.** | A | **44.** | C |
| **9.** | B | **18.** | C | **27.** | E | **36.** | B | **45.** | D |

The explanation for any question not given below will be found in the answer section to AB Practice Examination 1 on pages 474 to 482. Identical questions in Section I of Practice Examinations AB1 and BC1 have the same number. For example, explanations of the answers for Questions 1 through 6, not given below, are given in Section I of Examination AB1, answers 1 through 6, page 474.

**7.** D. The volume is given by $\lim_{k\to\infty} \pi \int_0^k e^{-2x}\,dx = \frac{\pi}{2}$.

**9.** B. Let $y = x^x$ and take logarithms. $\ln y = x \ln x = \frac{\ln x}{1/x}$. As $x \to 0^+$, this function has the indeterminate form $\infty/\infty$. Apply L'Hôpital's rule:

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{1/x}{-\,1/x^2} = \lim_{x\to 0^+} (-\,x) = 0$$

So $y \to e^0$ or 1.

**12.** A. We use the parts formula with $u = x$ and $dv = e^x\,dx$. Then $du = dx$ and $v = e^x$. We get

$$\left(xe^x - \int e^x\,dx\right)\Big|_0^1 = (xe^x - e^x)\Big|_0^1 = (e - e) - (0 - 1).$$

**15.** B. The arc length is given by the integral $\int_0^1 \sqrt{1 + 9x}$, which is equal to

$$\frac{1}{9} \cdot \frac{2}{3}(1 + 9x)^{3/2} \Big|_0^1 = \frac{2}{27}(10^{3/2} - 1).$$

**16.** E. Separating variables yields $\frac{dx}{x} = k\,dt$. Integrating, we get $\ln x = kt + C$. Since $x = 2$ when $t = 0$, $\ln 2 = C$. Then $\ln \frac{x}{2} = kt$. Using $x = 6$ when $t = 1$, it follows that $\ln 3 = k$.

**18.** C. $|\mathbf{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(\sec^2 t)^2 + (\sec t \tan t)^2}$. At $t = \frac{\pi}{6}$,

$$|\mathbf{v}| = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^4 + \left(\frac{2}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}\right)^2}.$$

**19.** C. At (0, 2), $\frac{dy}{dx} = 0$. With step size $\Delta x = \frac{1}{2}$, the first step moves us to $\left(\frac{1}{2}, 2\right)$, where $\frac{dy}{dx} = \frac{5}{2\frac{1}{2}} = 2$; so the next step produces $\left(1, 2 + \frac{1}{2}(2)\right)$.

**22.** D. Particular solutions appear to be branches of hyperbolas.

**23.** E. $\lim_{n\to\infty} \frac{n}{5n + 1} = \frac{1}{5} \neq 0$.

**24.** C. $\sqrt{1 - \sin^2 \theta} = \cos \theta$, $dx = \cos \theta \, d\theta$, $\sin^{-1} 0 = 0$, $\sin^{-1} 1 = \frac{\pi}{2}$.

**26.** B. Since $x + 2 = \sin t$ and $y = \cos^2 t$, we get

$$(x + 2)^2 + y = 1,$$

where $-3 \leqq x \leqq -1$ and $0 \leqq y \leqq 1$.

**Part B**

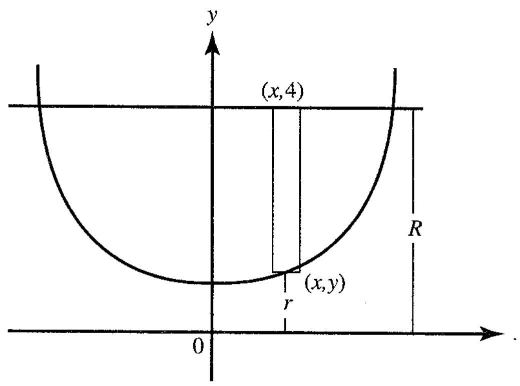
**29.** B. We set

$$\frac{x - 1}{x^2 + 3x + 2} = \frac{x - 1}{(x + 1)(x + 2)} = \frac{A}{x + 1} + \frac{B}{x + 2}.$$

$$x - 1 = A(x + 2) + B(x + 1);$$

$x = -2$ implies $-3 = -B$, or $B = 3$;

$x = -1$ implies $-2 = A$, or $A = -2$.




**30.** E. $S$ is the region bounded by $y = \sec x$ and $y = 4$.

We send region $S$ about the $x$-axis. Using washers, $\Delta V = \pi(R^2 - r^2)\ \Delta x$. Symmetry allows us to double the volume generated by the first-quadrant portion of $S$. So for $V$ we have

```
2πfnInt(16-(1/cos X)²,X,0,cos⁻¹.25)
```

which is 108.177.

**31.** A. We use the Ratio Test, page 346:

$$\lim_{n\to\infty}\left|\frac{(x-1)^{n+1}}{(n+1)!}\cdot\frac{n!}{(x-1)^n}\right| = \lim_{n\to\infty}\frac{1}{n+1}\left|x-1\right|,$$

which equals zero if $x \neq 1$. The series also converges if $x = 1$.

**32.** C. The absolute value function $f(x) = |x|$ is continuous at $x = 0$, but $f'(0)$ does not exist.

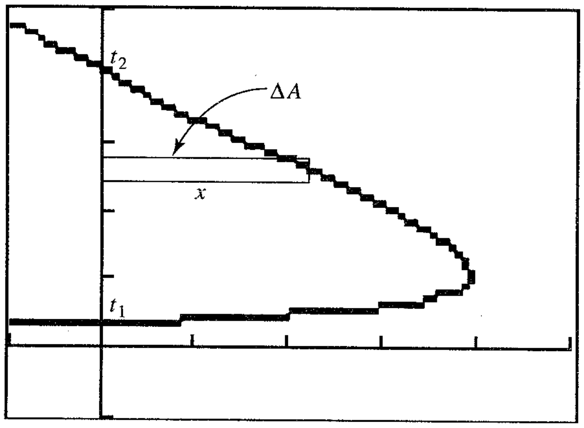
**33.** B. The Maclaurin series is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots$$

When we approximate the sum of an alternating series by using a finite number of terms, the error is less than the first term omitted. On the interval $-\pi \leqq x \leqq \pi$, the maximum error (numerically) occurs when $x = \pi$. Since

$$\frac{\pi^7}{7!} < 0.6 \quad \text{and} \quad \frac{\pi^9}{9!} < 0.09$$

four terms will suffice to assure no error greater than 0.1.




**34.** B. Letting $X_{1T}$ be $4-T^2$ and $Y_{1T}$ be $2^T$, graph the parametric equations in the following window: T in $[-3, 3]$, $(X_{1T}, Y_{1T})$ in $[-1, 5] \times [-1, 5]$. Now $\Delta A = x\Delta y$; the limits of integration are the two points where the curve cuts the $y$-axis, that is, where $x = 0$. In terms of $t$, these are $t_1 = -2$ and $t_2 = +2$. So

$$A = \int_{t=t_1}^{t=t_2} x\,dy = \int_{-2}^{2} (4 - t^2)2^t \ln 2\,dt$$

Using the calculator, enter either

```
A=fnINT((4-T²)(2^T ln 2),T,-2,2)
```

or

```
A=fnInt(X1T YIT ln 2,T,-2,2)
```

The answer is 8.916.

**36.** B. $\frac{dx}{dt} = 2$ and $\frac{dy}{dt} = -8t$. Use the fact that $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

**37.** A. Separate the variables to obtain $\frac{dy}{\cos^2 y} = \cos x\,dx$ and solve $\int \sec^2 y\,dy = \int \cos x\,dx$. Use the given condition to obtain the particular solution.

**39.** B. Solving by separation of variables we see that

$$\frac{dP}{1200 - P} = 0.16\,dt$$

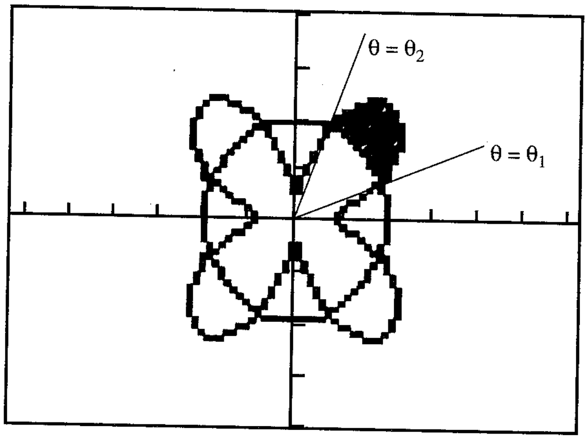
$$-\ln(1200 - P) = 0.16t + C$$

$$1200 - P = ce^{-0.16t}$$

Using $P(0) = 200$ we find $c = 1000$, so $P(x) = 1200 - 1000e^{-0.16t}$. Now $P(2) = 473.85$.

**40.** B. $y' = x + 2x \ln x$ and $y'' = 3 + 2 \ln x$.

**42.** C. The power series for ln $(1 - x)$, if $x < 1$, is $-x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots.$




**45.** D. Using the polar mode, let $r_1 = 3 \sin 2\theta$, $r_2 = 2$, and graph $r_1$ and $r_2$ in $[-6, 6] \times [-4, 4]$ for $\theta$ in $[0, 2\pi]$. The limits of integration are $\theta_1$ and $\theta_2$, where

$$\theta_1 = \text{solve } (r_1 - r_2, \theta, .5)$$

and

$$\theta_2 = \text{solve } (r_1 - r_2, \theta, 1)$$

Store $\theta_1$ as P, $\theta_2$ as Q.

To find the shaded area $A$ we subtract the area in the circle from that in the rose. So

$$A = \frac{1}{2}\int_P^Q r_1^2\, d\theta - \frac{1}{2}\int_P^Q r_2^2\, d\theta = \frac{1}{2}\int_P^Q (r_1^2 - r_2^2)\, d\theta$$

or, for the calculator,

$$.5\text{fnInt}(r_1^2 - r_2^2, \theta, P, Q)$$

which gives 1.328.

$\theta_2$ can be found quickly above by using Last Entry and then replacing .5 by 1.