44. $\int x \cos x^2\, dx$ equals

(A) $\sin x^2 + C$ (B) $2 \sin x^2 + C$ (C) $-\frac{1}{2}\sin x^2 + C$

(D) $\frac{1}{4}\cos^2 x^2 + C$ (E) $\frac{1}{2}\sin x^2 + C$

45. The rate at which a purification process can remove contaminants from a tank of water is proportional to the amount of contaminant remaining. If 20% of the contaminant can be removed during the first minute of the process and 98% must be removed to make the water safe, approximately how long will the decontamination process take?

(A) 2 min (B) 5 min (C) 18 min (D) 20 min (E) 40 min

Answers to AB Practice Examination 2: Section I

1.	C	10.	E	19.	B	28.	C	37.	D
2.	C	11.	C	20.	C	29.	B	38.	E
3.	B	12.	D	21.	A	30.	D	39.	E
4.	B	13.	A	22.	C	31.	E	40.	E
5.	B	14.	E	23.	B	32.	D	41.	B
6.	E	15.	E	24.	A	33.	D	42.	D
7.	A	16.	B	25.	D	34.	D	43.	E
8.	D	17.	B	26.	B	35.	D	44.	E
9.	B	18.	B	27.	B	36.	A	45.	C

Part A

1. C. Use the Rational Function Theorem on page 30.

2. C. Note that $\lim\limits_{h\to 0} \frac{\ln(2+h) - \ln 2}{h} = f'(2)$, where $f(x) = \ln x$.

3. B. Since $y' = -2xe^{-x^2}$, therefore

$$y'' = -2(x \cdot e^{-x^2} \cdot (-2x) + e^{-x^2}).$$

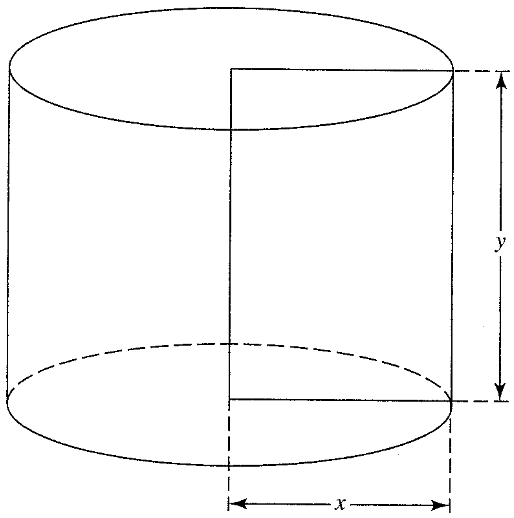
Replace x by 0.

4. B. $\frac{f(4) - f(1)}{4 - 1} = \frac{6 - 2}{4 - 1} = \frac{4}{3}.$

5. B. $h'(3) = g'(f(3)) \cdot f'(3) = g'(4) \cdot f'(3) = \frac{1}{2} \cdot 2.$

6. E. Since $f'(x)$ exists for all x, it must equal 0 for any x_0 for which f is a relative maximum, and it must also change sign from positive to negative as x increases through x_0. For the given derivative, no x satisfies both of these conditions.

7. A. The curve falls when $f'(x) < 0$ and is concave up when $f''(x) > 0$



8. D. See the figure. The volume V of the cylinder equals $\pi x^2 y$, where $2x+2y = 18$. So, $V = \pi x^2(9 - x)$ and $V' = \pi(18x - 3x^2)$. Since $x = 6$ yields maximum volume, the area of the rectangle, xy, equals 18.

9. B. Since $\frac{dr}{dt} = k$, a positive constant, $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2 k = cr^2$, where c is a positive constant. Then $\frac{d^2V}{dt^2} = 2cr\frac{dr}{dt} = 2crk$, which is also positive.

10. E. Acceleration is the derivative (the slope) of velocity v; v is steepest on $8 < t < 9$.

11. C. Velocity v is the derivative of position; $v > 0$ until $t = 6$; $v < 0$ thereafter.

12. D. From $t = 5$ to $t = 8$ the displacement can be found by determining the areas of two triangles: $\frac{1}{2}(1)(2) + \frac{1}{2}(2)(-4) = -3$. Thus if K is its position at $t = 5$, then $K - 3 = 10$ at $t = 8$.

13. A. Evaluate $\frac{1}{4}\sin^4 \alpha \Big|_{\pi/4}^{\pi/2}$.

14. E. Evaluating $\frac{1}{3 - e^x}\Big|_0^1$ yields $\frac{e - 1}{2(3 - e)}$.

15. E. Since $f'(x) = 1 - \frac{c}{x^2}$, it equals 0 for $x = \pm\sqrt{c}$. When $x = 3$, $c = 9$; this yields a minimum since $f''(3) > 0$.

16. B. $f(x) = (1 - e^{-x})$ is increasing and concave downward.

17. B. Implicit differentiation yields $2yy' = 1 - 3x^2$; so $\frac{dy}{dx} = \frac{1 - 3x^2}{2y}$. At a vertical tangent, $\frac{dy}{dx}$ is undefined; y must therefore equal 0. The original equation with $y = 0$ is $0 = x - x^3$, which has 3 solutions.

18. B. Let $t = x - 1$.

19. B. The required area, A, is given by the integral

$$2\int_0^1 \left(4 - \frac{4}{1 + x^2}\right) dx = 2(4x - 4\tan^{-1} x)\Big|_0^1 = 2\left(4 - 4 \cdot \frac{\pi}{4}\right).$$

20. C. The average value equals $\frac{1}{3}\left(\frac{t^3}{6} - \frac{t^4}{12}\right)\Big|_{-2}^{1}$.

21. A. Solve the differential equation $\frac{dy}{dx} = 2y$ by separation of variables: $\frac{dy}{y} = 2dx$ yields $y = ce^{2x}$. The initial condition yields $1 = ce^{2 \cdot 2}$; so $c = e^{-4}$ and $y = e^{2x-4}$.

22. C. Changes in values of f'' show that f''' is constant.

23. B. By implicit differentiation, $3x^2 + x \sec^2 y \frac{dy}{dx} + \tan y = 0$. At (3,0), $\frac{dy}{dx} = -9$; so the equation of the tangent line at (3,0) is $y = -9(x-3)$.

24. A. $(h^{1/2})' = 2h'$ implies $\frac{1}{2} h^{-1/2} = 2$.

25. D. The graph shown has the following characteristics: it has no x-intercepts; the y-intercept is -2; it has vertical asymptotes $x = 1$ and $x = -1$; and it has the x-axis as horizontal asymptote.

26. B. Since $\lim_{x\to 1} f(x) = 1$, to render $f(x)$ continuous at $x = 1$ we must define $f(1)$ to be 1.

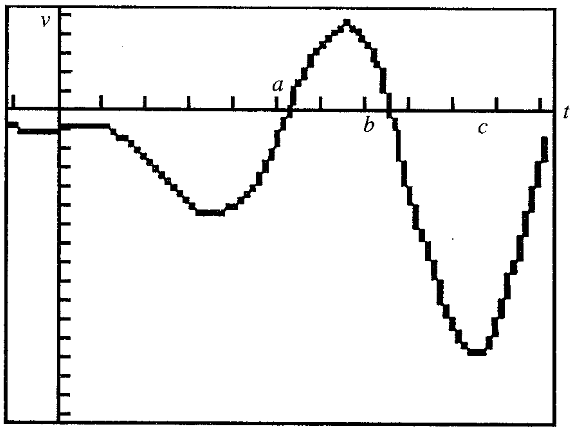
27. B. $f'(x) = 15x^4 - 30x^2$; $f''(x) = 60x^3 - 60x = 60x(x + 1)(x - 1)$; this equals 0 when $x = -1$, 0, or 1. Here are the signs within the intervals:

28. C. Note that $f'(x) = \frac{4 + x}{x^2 + 4}$, so f has a critical value at $x = -4$. As x passes through -4, the sign of f' changes from $-$ to $+$, so f has a local minimum at $x = -4$.

Part B

29. B. We are given that (1) $f'(a) > 0$; (2) $f''(a) < 0$; and (3) $G'(a) < 0$. Since $G'(x) = 2f(x) \cdot f'(x)$, therefore $G'(a) = 2f(a) \cdot f'(a)$. Conditions (1) and (3) imply that (4) $f(a) < 0$. Since $G''(x) = 2[f(x) \cdot f''(x) + (f'(x))^2]$, therefore $G''(a) = 2[f(a)f''(a) + (f'(a))^2]$. Then the sign of $G''(a)$ is $2[(-) \cdot (-) + (+)]$ or positive, where the minus signs in the parentheses follow from conditions (4) and (2).

30. D. $f'(2.1) \simeq \dfrac{f(2.2) - f(2.0)}{2.2 - 2.0}$.



31. E. The calculator figure maps velocity Y_1 against time. Speed is the absolute value of velocity. The greatest deviation from $Y_1 = 0$ is at $t = c$. With a calculator, using the `[minimum]` option, we get $c = 9.538$.

32. D. f' changes from increasing to decreasing, so f'' changes from positive to negative.

33. D. $H(3) = \int_0^3 f(t)\,dt = -2$; $H'(3) = f(3) = 2$.

34. D. $T = \frac{1}{2}[30 + 2(22) + 2(12) + 0]$.

35. D. $\int_0^8 R(x)\,dx = 166.396$.

36. A. Selecting an answer for this question from your calculator graph is unwise. In some windows the graph may appear continuous; in others there may seem to be cusps, or a vertical asymptote. If we put the calculator aside, we find

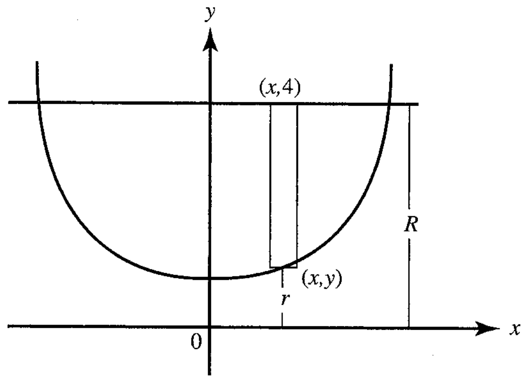
$$\lim_{x\to 1^+}\left(\arctan\left(\frac{1}{\ln x}\right)\right) = \frac{\pi}{2} \text{ and } \lim_{x\to 1^-}\left(\arctan\left(\frac{1}{\ln x}\right)\right) = -\frac{\pi}{2}$$

These indicate the presence of a jump discontinuity at $x = 1$.

37. D. $\frac{d}{du}\int_0^u \frac{1}{1+x^2}\,dx = \frac{1}{1+u^2}$. When $u = t^2$,

$$\frac{d}{dt}\int_0^u \frac{1}{1+x^2}\,dx = \frac{1}{1+u^2}\,\frac{du}{dt} = \frac{1}{1+t^4}(2t).$$

38. E. $\int(\sqrt{x} - 2)x^2dx = \int(x^{5/2} - 2x^2)dx = \frac{2}{7}x^{7/2} - \frac{2}{3}x^3 + C.$



39. E. S is the region bounded by y sec x and $y = 4$.

We send region S about the x-axis. Using washers, $\Delta V = \pi(R^2 - r^2)\,\Delta x$. Symmetry allows us to double the volume generated by the first-quadrant of S. So for V we have

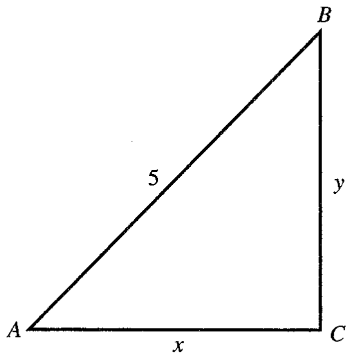
```
2πfnInt(16-(1/cos X)²,X,0,cos⁻¹.25)
```

which is 108.177.

40. E. Use the quotient rule (formula (6) on page 46).

41. B. $g'(y) = \frac{1}{f'(x)} = \frac{1}{5x^4}$. To find $g'(0)$, we seek x such that $f(x) = 0$. By inspection,

$x = -1$, so $g'(0) = \frac{1}{5(-1)^4} = \frac{1}{5}$.



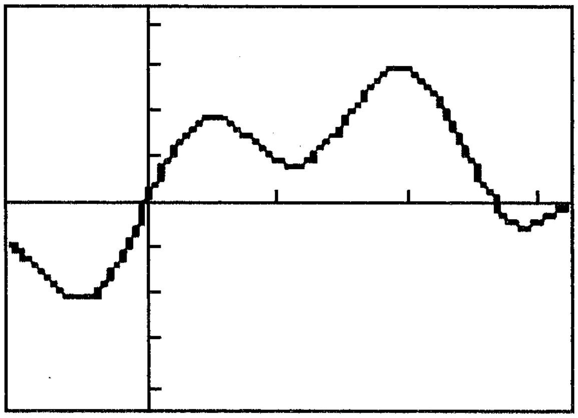
42. D. See the figure. It is given that $\frac{dx}{dt} = -2$; we want $\frac{dA}{dt}$, where $A = \frac{1}{2}xy$.

$$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right) = \frac{1}{2}\left[3 \cdot \frac{dy}{dt} + y \cdot (-2)\right].$$

Since $y^2 = 25 - x^2$, it follows that $2y\frac{dy}{dt} = -2x\frac{dx}{dt}$ and when $x = 3$,

$$y = 4 \quad \text{and} \quad \frac{dy}{dt} = \frac{3}{2}.$$

Then $\frac{dA}{dt} = -\frac{7}{4}$.



43. E. Since $f(0) = f(\pi) = 0$ and f is both continuous and differentiable, Rolle's theorem predicts at least one c in the interval such that $f'(c) = 0$. Key in

```
Y1=2sin X+sin 4X
```

and graph `Y1` in `[-1,π]×[-4,4]`. We see four points in $[0,\pi]$ where the tangent is horizontal.

44. E. Use formula (5) on page 150 with $u = x^2$; $du = 2x\,dx$.

45. C. If $Q(t)$ is the amount of contaminant in the tank at time t and Q_0 is the initial amount, then

$$\frac{dQ}{dt} = kQ \quad \text{and} \quad Q(t) = Q_0 e^{kt}$$

Since $Q(1) = 0.8Q_0$, $0.8Q_0 = Q_0 e^{k \cdot 1}$, $0.8 = e^k$, and

$$Q(t) = Q_0(0.8)^t$$

We seek t when $Q(t) = 0.02Q_0$. Thus,

$$0.02Q_0 = Q_0(0.8)^t.$$

and

$$t \simeq 17.53 \text{ min.}$$