AB Practice Examination 2

Section I

Part A†

(See instructions, page 467. Answers begin on page 491.)

1. $\lim_{x\to\infty} \dfrac{20x^2 - 13x + 5}{5 - 4x^3}$ is

(A) -5 **(B)** ∞ **(C)** 0 **(D)** 5 **(E)** 1

2. $\lim_{h\to 0} \dfrac{\ln(2+h) - \ln 2}{h}$ is

(A) 0 **(B)** $\ln 2$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{1}{\ln 2}$ **(E)** ∞

3. If $y = e^{-x^2}$, then $y''(0)$ equals

(A) 2 **(B)** -2 **(C)** $\dfrac{2}{e}$ **(D)** 0 **(E)** -4

The table shown is for Questions 4 and 5. The differentiable functions f and g have the values shown.

x	f	f'	g	g'
1	2	$\frac{1}{2}$	-3	5
2	3	1	0	4
3	4	2	2	3
4	6	4	3	$\frac{1}{2}$

4. The average rate of change of function f on [1,4] is

(A) 7/6 **(B)** 4/3 **(C)** 15/8 **(D)** 9/4 **(E)** 8/3

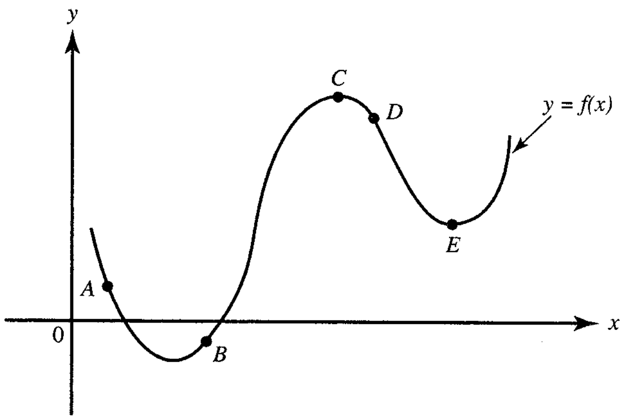
†Beginning in May 1998, 55 minutes will be allowed for Part A.

5. If $h(x) = g(f(x))$ then $h'(3) =$
(A) 1/2 **(B)** 1 **(C)** 4 **(D)** 6 **(E)** 9

6. The derivative of a function f is given for all x by

$$f'(x) = x^2(x + 1)^3(x - 4)^2.$$

The set of x for which f is a relative maximum is
(A) $\{0, -1, 4\}$ **(B)** $\{-1\}$ **(C)** $\{0, 4\}$ **(D)** $\{1\}$
(E) none of these



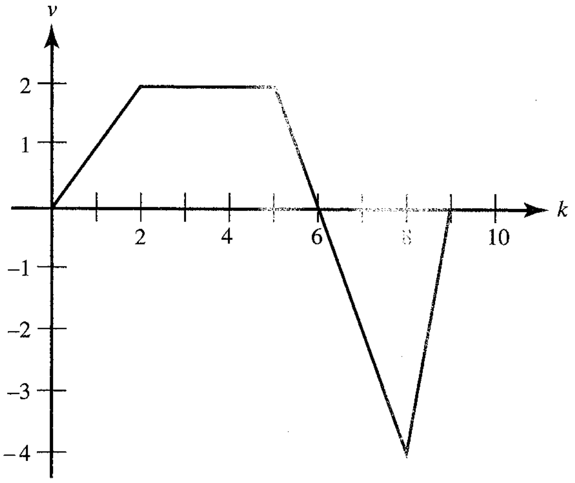
7. At which point on the graph of $y = f(x)$ shown above is $f'(x) < 0$ and $f''(x) > 0$?
(A) A **(B)** B **(C)** C **(D)** D **(E)** E

8. A rectangle of perimeter 18 in. is rotated about one of its sides to generate a right circular cylinder. The rectangle which generates the cylinder of largest volume has area, in square inches, of

(A) 14 **(B)** 20 **(C)** $\frac{81}{4}$ **(D)** 18 **(E)** $\frac{77}{4}$

9. If the radius r of a sphere is increasing at a constant rate, then the rate of increase of the volume of the sphere is
(A) constant **(B)** increasing **(C)** decreasing
(D) increasing for $r < 1$ and decreasing for $r > 1$
(E) decreasing for $r < 1$ and increasing for $r > 1$

For Questions 10, 11, and 12, the graph shows the velocity of an object moving along a line, for $0 \leqslant t \leqslant 9$.



10. At what time does the object attain its maximum acceleration?
(A) $2<t<5$ **(B)** $5<t<8$ **(C)** $t=6$ **(D)** $t=8$ **(E)** $8<t<9$

11. The object is farthest from the starting point at $t =$
(A) 2 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 9

12. At $t = 8$, the object was at position $x = 10$. At $t = 5$, the position was $x =$
(A) −5 **(B)** 5 **(C)** 7 **(D)** 13 **(E)** 15

13. $\int_{\pi/4}^{\pi/2} \sin^3 \alpha \cos \alpha \, d\alpha$ is equal to
(A) $\frac{3}{16}$ **(B)** $\frac{1}{8}$ **(C)** $-\frac{1}{8}$ **(D)** $-\frac{3}{16}$ **(E)** $\frac{3}{4}$

14. $\int_0^1 \frac{e^x}{(3 - e^x)^2} \, dx$ equals
(A) $3 \ln (e-3)$ **(B)** 1 **(C)** $\frac{1}{3 - e}$ **(D)** $\frac{e - 2}{3 - e}$ **(E)** none of these

15. The value of c for which $f(x) = x + \frac{c}{x}$ has a local minimum at $x = 3$ is
(A) −9 **(B)** −6 **(C)** −3 **(D)** 6 **(E)** 9

16. If $\int_0^1 (1 - e^{-x}) \, dx$ is approximated using Riemann sums and the same number of subdivisions, and if L, R, M, and T denote respectively Left, Right, Midpoint, and Trapezoid sums, then it follows that
(A) $L \leqslant R \leqslant M \leqslant T$ **(B)** $L \leqslant T \leqslant M \leqslant R$ **(C)** $L \geqslant R \geqslant M \geqslant T$
(D) $L \leqslant M \leqslant T \leqslant R$ **(E)** none of these

17. The number of vertical tangents to the graph of $y^2 = x - x^3$ is
(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

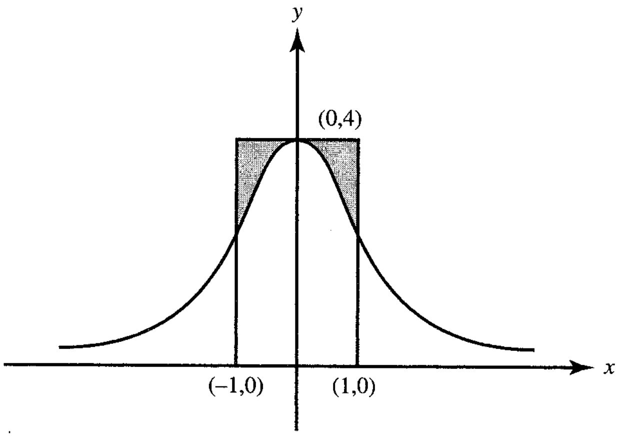
18. $\int_0^6 f(x-1)\,dx =$

(A) $\int_{-1}^{7} f(x)\,dx$ **(B)** $\int_{-1}^{5} f(x)\,dx$ **(C)** $\int_{-1}^{5} f(x+1)\,dx$

(D) $\int_{1}^{5} f(x)\,dx$ **(E)** $\int_{1}^{7} f(x)\,dx.$

19. The equation of the curve shown below is $y = \dfrac{4}{1+x^2}$. What does the area of the shaded region equal?

(A) $4 - \dfrac{\pi}{4}$ **(B)** $8 - 2\pi$ **(C)** $8 - \pi$ **(D)** $8 - \dfrac{\pi}{2}$ **(E)** $2\pi - 4$



20. The average value of $\frac{1}{2}t^2 - \frac{1}{3}t^3$ over the interval $-2 \leq t \leq 1$ is

(A) $\dfrac{1}{36}$ **(B)** $\dfrac{1}{12}$ **(C)** $\dfrac{11}{12}$ **(D)** 2 **(E)** $\dfrac{33}{12}$

21. If $f'(x) = 2f(x)$ and $f(2) = 1$, then $f(x) =$

(A) e^{2x-4} **(B)** $e^{2x}+1-e^4$ **(C)** e^{4-2x} **(D)** e^{2x+1} **(E)** e^{x-2}

22. The table below shows values of $f''(x)$ for various values of x:

x	−1	0	1	2	3
$f''(x)$	−4	−1	2	5	8

The function f could be

(A) a linear function **(B)** a quadratic function

(C) a cubic function **(D)** a fourth-degree function

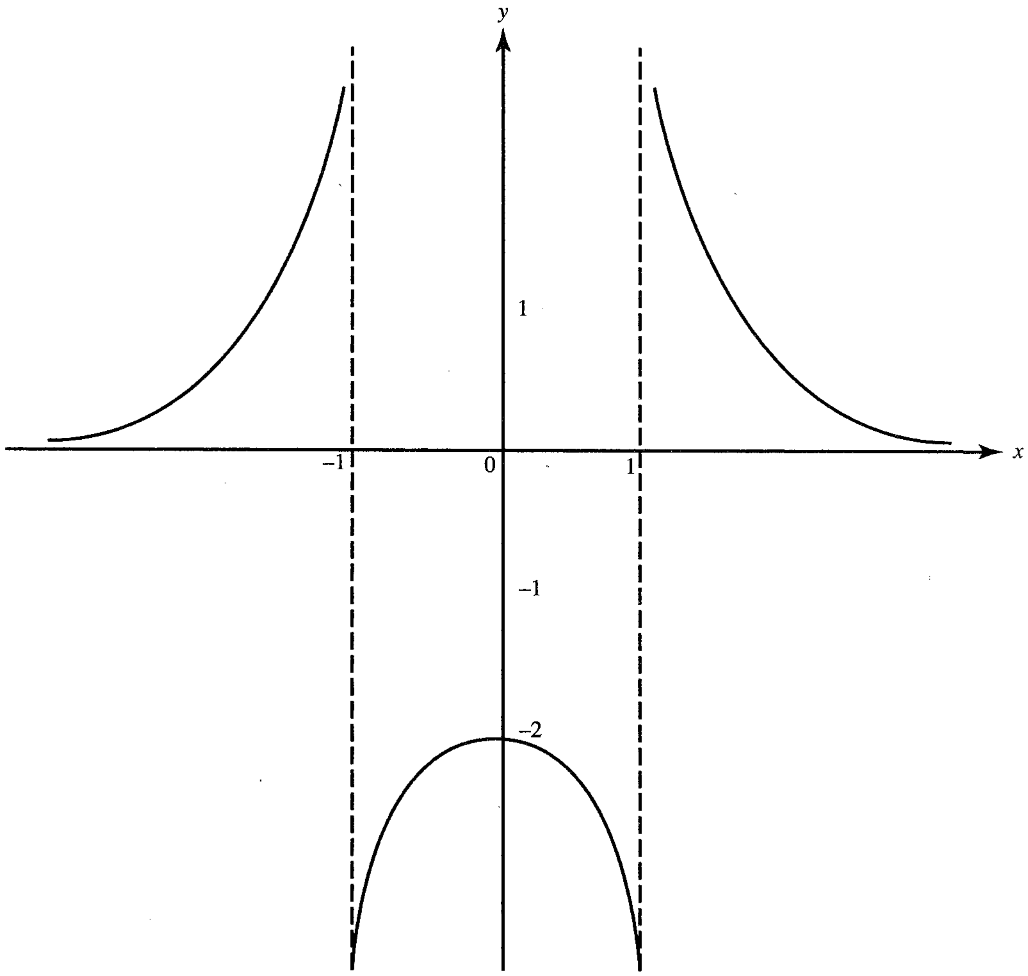
(E) an exponential function

23. The curve $x^3 + x\tan y = 27$ passes through (3,0). Use local linearization to estimate the value of y at $x = 3.1$. The value is

(A) −2.7 **(B)** −0.9 **(C)** 0 **(D)** 0.1 **(E)** 3.0

24. At what value of h is the rate of increase of $\sqrt{h}$ twice the rate of increase of h?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{4}$ **(C)** 1 **(D)** 2 **(E)** 4



25. If the graph of a function is as shown above, then the function $f(x)$ could be given by which of the following?

(A) $f(x) = \frac{x+2}{x^2-1}$ **(B)** $f(x) = \frac{1}{1-x^2}$ **(C)** $f(x) = \frac{x^2-1}{x^2+1}$

(D) $f(x) = \frac{2}{x^2-1}$ **(E)** $f(x) = \frac{2}{1-x^2}$

26. A function $f(x)$ equals $\frac{x^2-x}{x-1}$ for all x except $x = 1$. In order that the function be continuous at $x = 1$, the value of $f(1)$ must be

(A) 0 **(B)** 1 **(C)** 2 **(D)** ∞ **(E)** none of these

27. The number of inflection points of $f(x) = 3x^5 - 10x^3$ is

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

28. Suppose $f(x) = \int_0^x \frac{4+t}{t^2+4}\,dt$. It follows that

(A) f increases for all x (B) f increases only if $x < -4$
(C) f has a local min at $x = -4$ (D) f has a local max at $x = -4$
(E) f has no critical points

Part B†

(See instructions, page 471. Answers begin on page 494.)

29. Let $G(x) = [f(x)]^2$. At $x = a$, f is increasing and concave downward, while G is decreasing. Which describes G at $x = a$?

(A) concave downward (B) concave upward (C) linear
(D) point of inflection (E) none of these

30. A differentiable function has values shown in this table:

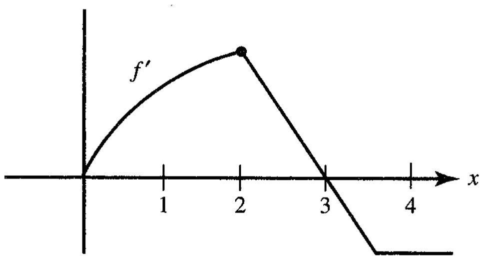
x	2.0	2.2	2.4	2.6	2.8	3.0
$f'(x)$	1.39	1.73	2.10	2.48	2.88	3.30

Estimate $f'(2.1)$.

(A) 0.34 (B) 0.59 (C) 1.56 (D) 1.70 (E) 1.91

31. An object moving along a line has velocity $v(t) = t\cos t - \ln(t+2)$, where $0 \le t \le 10$. The object achieves its maximum speed when $t =$

(A) 3.743 (B) 5.107 (C) 6.419 (D) 7.550 (E) 9.538



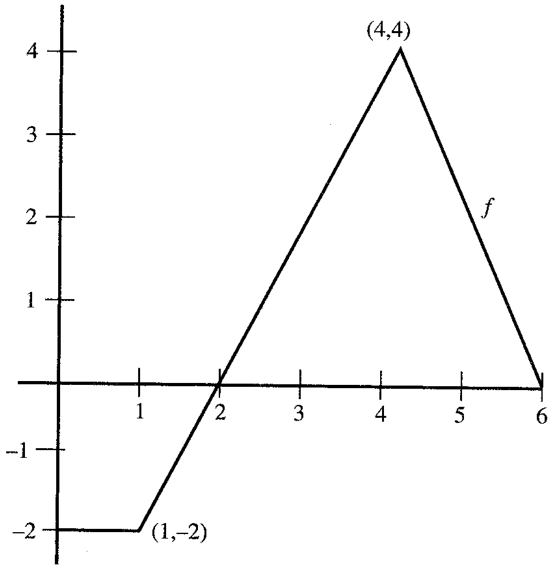
f' consists of a quarter-circle and two line-segments

32. The graph of f' is shown above. At $x = 2$ which of the following statements is true?

(A) f is not continuous
(B) f is continuous but not differentiable
(C) f has a relative maximum
(D) f has a point of inflection
(E) none of these

†Beginning in May 1998, 50 minutes will be allowed for Part B.

33. Let $H(x) = \int_0^x f(t)\,dt$, where f is the function whose graph appears here:



The local linearization of $H(x)$ near $x = 3$ is $H(x) \simeq$

(A) $-2x + 8$ **(B)** $2x - 4$ **(C)** $-2x + 4$ **(D)** $2x - 8$ **(E)** $2x - 2$

34. The table shows the speed of an object in feet per second during a 3-second period. Estimate the distance the object travels, using the trapezoid method:

time (sec)	0	1	2	3
speed (ft/sec)	30	22	12	0

(A) 34 ft **(B)** 45 ft **(C)** 48 ft **(D)** 49 ft **(E)** 64 ft

35. In a marathon, when the winner crosses the finish line many runners are still on the course, some quite far behind. If the density of runners x miles from the finish line is given by $R(x) = 20[1 - \cos(1 + .03x^2)]$ runners per mile, how many are within 8 miles of the finish line?

(A) 30 **(B)** 145 **(C)** 157 **(D)** 166 **(E)** 195

36. Which best describes the behavior of the function $y = \arctan\left(\frac{1}{\ln x}\right)$ at $x = 1$?

(A) It has a jump discontinuity.
(B) It has an infinite discontinuity.
(C) It has a removable discontinuity.
(D) It is both continuous and differentiable.
(E) It is continuous but not differentiable.

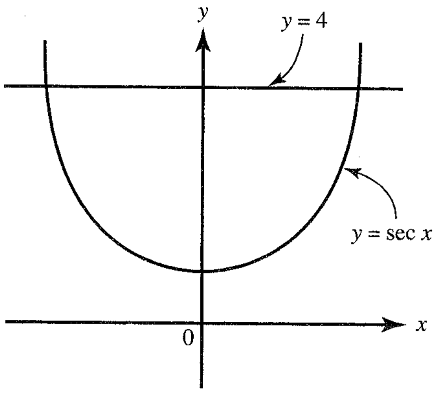
37. If $f(t) = \int_0^{t^2} \frac{1}{1 + x^2}\,dx$, then $f'(t)$ equals

(A) $\frac{1}{1 + t^2}$ **(B)** $\frac{2t}{1 + t^2}$ **(C)** $\frac{1}{1 + t^4}$ **(D)** $\frac{2t}{1 + t^4}$ **(E)** $\tan^{-1} t^2$

38. $\int (\sqrt{x} - 2)\, x^2\, dx =$

(A) $\frac{2}{3}x^{3/2} - 2x + C$ **(B)** $\frac{5}{2}x^{3/2} - 4x + C$ **(C)** $\frac{2}{3}x^{3/2} - 2x + \frac{x^3}{3} + C$

(D) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^3 + C$ **(E)** $\frac{2}{7}x^{7/2} - \frac{2}{3}x^3 + C$



39. The region S in the figure shown is bounded by $y = \sec x$ and $y = 4$. What is the volume of the solid formed when S is rotated about the x-axis?

(A) 0.304 **(B)** 39.867 **(C)** 53.126 **(D)** 54.088 **(E)** 108.177

40. If $y = \frac{x-3}{2-5x}$, then $\frac{dy}{dx}$ equals

(A) $\frac{17 - 10x}{(2-5x)^2}$ **(B)** $\frac{13}{(2-5x)^2}$ **(C)** $\frac{x-3}{(2-5x)^2}$ **(D)** $\frac{17}{(2-5x)^2}$

(E) $\frac{-13}{(2-5x)^2}$

41. Let $f(x) = x^5 + 1$ and let g be the inverse function of f. What is the value of $g'(0)$?

(A) -1 **(B)** $\frac{1}{5}$ **(C)** 1 **(D)** $g'(0)$ does not exist.

(E) It cannot be determined from the given information.

42. The hypotenuse AB of a right triangle ABC is 5 ft, and one leg, AC, is decreasing at the rate of 2 ft/sec. The rate, in square feet per second, at which the area is changing when $AC = 3$ is

(A) $\frac{25}{4}$ **(B)** $\frac{7}{4}$ **(C)** $-\frac{3}{2}$ **(D)** $-\frac{7}{4}$ **(E)** $-\frac{7}{2}$

43. At how many points on the interval $[0,\pi]$ does $f(x) = 2\sin x + \sin 4x$ satisfy the Mean Value Theorem?

(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

44. $\int x \cos x^2 \, dx$ equals

(A) $\sin x^2 + C$ **(B)** $2 \sin x^2 + C$ **(C)** $-\frac{1}{2} \sin x^2 + C$

(D) $\frac{1}{4} \cos^2 x^2 + C$ **(E)** $\frac{1}{2} \sin x^2 + C$

45. The rate at which a purification process can remove contaminants from a tank of water is proportional to the amount of contaminant remaining. If 20% of the contaminant can be removed during the first minute of the process and 98% must be removed to make the water safe, approximately how long will the decontamination process take?

(A) 2 min **(B)** 5 min **(C)** 18 min **(D)** 20 min **(E)** 40 min

Answers to AB Practice Examination 2: Section I

1.	C	**10.**	E	**19.**	B	**28.**	C	**37.**	D
2.	C	**11.**	C	**20.**	C	**29.**	B	**38.**	E
3.	B	**12.**	D	**21.**	A	**30.**	D	**39.**	E
4.	B	**13.**	A	**22.**	C	**31.**	E	**40.**	E
5.	B	**14.**	E	**23.**	B	**32.**	D	**41.**	B
6.	E	**15.**	E	**24.**	A	**33.**	D	**42.**	D
7.	A	**16.**	B	**25.**	D	**34.**	D	**43.**	E
8.	D	**17.**	B	**26.**	B	**35.**	D	**44.**	E
9.	B	**18.**	B	**27.**	B	**36.**	A	**45.**	C