AB Practice Examination 1

Section I

Part A†

The use of calculators is not permitted for this part of the examination.
(Answers are given on page 474).
There are 28 questions in Part A. To make up for possible guessing, the grade on this part is determined by subtracting one-fourth of the number of wrong answers from the number answered correctly.

1. $\lim_{x\to\infty} \dfrac{3x^2 - 4}{2 - 7x - x^2}$ is

(A) 3 **(B)** 1 **(C)** -3 **(D)** ∞ **(E)** 0

2. $\lim_{h\to 0} \dfrac{\cos\left(\frac{\pi}{2} + h\right)}{h}$ is

(A) 1 **(B)** nonexistent **(C)** 0 **(D)** -1 **(E)** none of these

3. If, for all x, $f'(x) = (x - 2)^4(x - 1)^3$, it follows that the function f has
(A) a relative minimum at $x = 1$
(B) a relative maximum at $x = 1$
(C) both a relative minimum at $x = 1$ and a relative maximum at $x = 2$
(D) neither a relative maximum nor a relative minimum
(E) relative minima at $x = 1$ and at $x = 2$

4. Let $(Fx) = \int_0^x \dfrac{10}{1 + e^t}\, dt$. Which of the following statements are true?

I. $F'(0) = 5$ **II.** $F(2) < F(6)$ **III.** F is concave upward

(A) I only **(B)** II only **(C)** III only
(D) I and II **(E)** I and III

5. If $f(x) = 10^x$ and $10^{1.04} \approx 10.96$, which is closest to $f'(1)$?
(A) 0.24 **(B)** 0.92 **(C)** 0.96 **(D)** 10.5 **(E)** 24

6. If f is differentiable, we can use the line tangent to f at $x = a$ to approximate values of f near $x = a$. Suppose this method always underestimates the correct values. If so, then at $x = a$, f must be
(A) positive **(B)** increasing **(C)** decreasing
(D) concave upward **(E)** concave downward

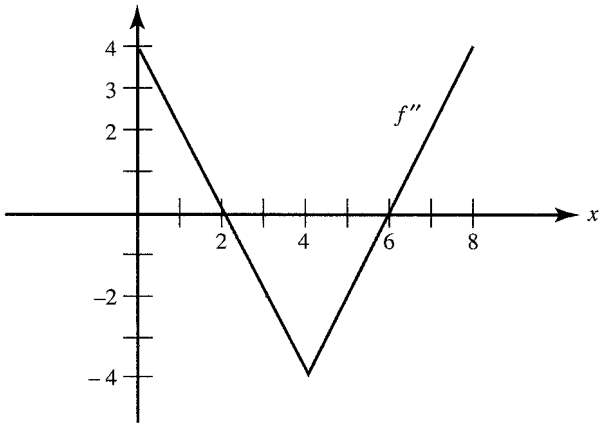
7. If $f(x) = \cos x \sin 3x$, then $f'\left(\frac{\pi}{6}\right)$ is equal to

(A) $\frac{1}{2}$ **(B)** $-\frac{\sqrt{3}}{2}$ **(C)** 0 **(D)** 1 **(E)** $-\frac{1}{2}$

8. $\int_0^1 \frac{x\,dx}{x^2 + 1}$ is equal to

(A) $\frac{\pi}{4}$ **(B)** $\ln \sqrt{2}$ **(C)** $\frac{1}{2}(\ln 2 - 1)$ **(D)** $\frac{3}{2}$ **(E)** $\ln 2$

9. The graph of f'' is shown below. If $f'(1) = 0$, then $f'(x) = 0$ at $x =$
(A) 0 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 7



The table shows the values of differentiable functions f and g, for Questions 10 and 11.

x	f	f'	g	g'
1	2	$\frac{1}{2}$	−3	5
2	3	1	0	4
3	4	2	2	3
4	6	4	3	$\frac{1}{2}$

10. If $P(x) = g^2(x)$, then $P'(3)$ equals

(A) 4 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 18

11. If $H(x) = f^{-1}(x)$, then $H'(3)$ equals

(A) $-\frac{1}{16}$ **(B)** $-\frac{1}{8}$ **(C)** $-\frac{1}{2}$ **(D)** $\frac{1}{2}$ **(E)** 1

12. The total area of the region bounded by the graph of $y = x\sqrt{1 - x^2}$ and the x-axis is

(A) $\frac{1}{3}$ **(B)** $\frac{1}{3}\sqrt{2}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** 1

13. The curve of $y = \frac{1 - x}{x - 3}$ is concave up when

(A) $x > 3$ **(B)** $1 < x < 3$ **(C)** $x > 1$ **(D)** $x < 1$ **(E)** $x < 3$

14. The area of the largest isosceles triangle that can be drawn with one vertex at the origin and with the others on a line parallel to and above the x-axis and on the curve $y = 27 - x^2$ is

(A) 108 **(B)** 27 **(C)** $12\sqrt{3}$ **(D)** 3 **(E)** $24\sqrt{3}$

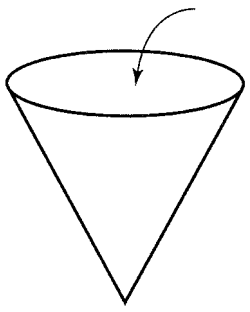
15. The average (mean) value of $\tan x$ on the interval from $x = 0$ to $x = \frac{\pi}{3}$ is

(A) $\ln 2$ **(B)** $\frac{3}{\pi}\ln 2$ **(C)** $\ln\frac{1}{2}$ **(D)** $\frac{9}{\pi}$ **(E)** $\frac{\sqrt{3}}{2}$

16. Using the substitution $x = 2t + 1$, which of the following is equivalent to $\int_0^3 \sqrt[4]{2t + 1}\ dt$?

(A) $\int_0^3 \sqrt[4]{x}\ dx$ **(B)** $\frac{1}{2}\int_0^3 \sqrt[4]{x}\ dx$ **(C)** $\frac{1}{2}\int_{-\frac{1}{2}}^1 \sqrt[4]{x}\ dx$ **(D)** $\int_1^7 \frac{1}{2}\sqrt[4]{x}\ dx$

(E) $2\int_1^7 \sqrt[4]{x}\ dx$



17. Water is poured at a constant rate into a conical reservoir (shown in the figure). If the depth of the water is graphed as a function of time, the graph is

(A) decreasing **(B)** constant
(C) linear **(D)** concave upward
(E) concave downward

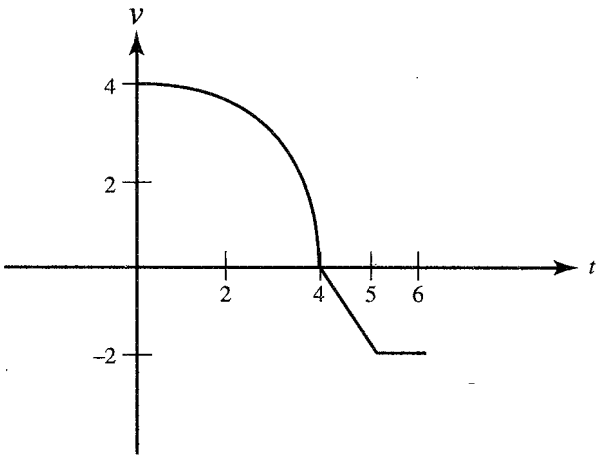
18. If $f(x) = \begin{cases} x^2 & \text{for } x \leq 1 \\ 2x - 1 & \text{for } x > 1 \end{cases}$, then

(A) $f(x)$ is not continuous at $x = 1$
(B) $f(x)$ is continuous at $x = 1$ but $f'(1)$ does not exist
(C) $f'(1)$ exists and equals 1
(D) $f'(1) = 2$
(E) $\lim_{x \to 1} f(x)$ does not exist

19. $\lim_{x \to 2^-} \dfrac{|x - 2|}{x - 2}$ is

(A) $-\infty$ **(B)** -1 **(C)** 1 **(D)** ∞ **(E)** nonexistent

The graph shown is for Questions 20 and 21. It consists of a quarter-circle and two line segments, and represents the velocity of an object during the six-second interval.



20. The object's average speed during the six-second interval is

(A) $\dfrac{4\pi + 3}{6}$ **(B)** $\dfrac{4\pi - 3}{6}$ **(C)** -1 **(D)** $-\dfrac{1}{3}$ **(E)** 1

21. The object's acceleration at $t = 2$ is

(A) -1 **(B)** $-\dfrac{1}{2}$ **(C)** $-\dfrac{1}{3}$ **(D)** $-\dfrac{1}{\sqrt{3}}$ **(E)** $-\sqrt{3}$

22. An integral for the volume obtained by revolving, around the x-axis, the region bounded by $y = 2x - x^2$ and the x-axis is

(A) $4\pi \int_0^1 x(2x - x^2)\,dx$ **(B)** $2\pi \int_0^2 x(2x - x^2)\,dx$

(C) $\pi \int_0^2 (2x - x^2)^2\,dx$ **(D)** $\pi \int_0^2 [(2x)^2 - (x^2)^2]\,dx$ **(E)** none of the above

23. A bank account earns interest compounded continuously at a rate of 6% per year. How long will it take, approximately, for the account to triple in value?

(A) 3 years **(B)** 9 years **(C)** 18 years **(D)** 33 years **(E)** 50 years

24. In the following, $L(n)$, $R(n)$, $M(n)$, and $T(n)$ denote respectively Left, Right, Midpoint, and Trapezoidal sums with n subdivisions. Which of the following is equal exactly to $\int_{-1}^{1} |x|\,dx$?

(A) $L(1)$ **(B)** $L(2)$ **(C)** $R(1)$ **(D)** $M(1)$ **(E)** none of these

25. $\int_a^x g(t)\,dt - \int_b^x g(t)\,dt$ is equal to the constant

(A) 0 **(B)** $b - a$ **(C)** $a - b$ **(D)** $\int_a^b g(t)\,dt$ **(E)** $g(b) - g(a)$

26. The solution of the differential equation $\frac{dy}{dx} = 2xy^2$ for which $y = -1$ when $x = 1$ is

(A) $y = -\frac{1}{x^2}$ **(B)** $\ln y^2 = x^2 - 1$ **(C)** $\frac{y^3}{3} = x^2 - \frac{4}{3}$

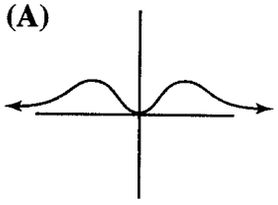
(D) $y = -\frac{1}{x}$ **(E)** none of these

27. The base of a solid is the region bounded by the parabola $y^2 = 4x$ and the line $x = 2$. Each plane section perpendicular to the x-axis is a square. The volume of the solid is

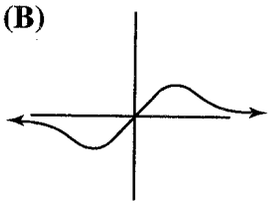
(A) 6 **(B)** 8 **(C)** 10 **(D)** 16 **(E)** 32

28. Which of the following could be the graph of $y = \frac{x^2}{e^x}$?

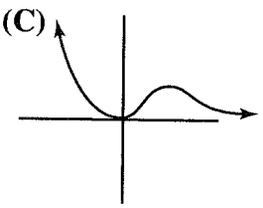
(A)



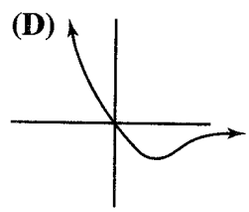
(B)



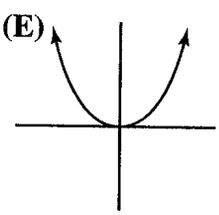
(C)



(D)



(E)



Part B†

Some questions in this part of the examination require the use of a graphing calculator. (Answers are given on page 474.)

There are 17 questions in Part B. The penalty for guessing on this part is the same as that for Part A. If the exact numerical value of the correct answer is not listed as a choice, select the choice that is closest to the exact numerical answer.

29. If $f(3) = 8$ and $f'(3) = -4$ then $f(3.02)$ is approximately

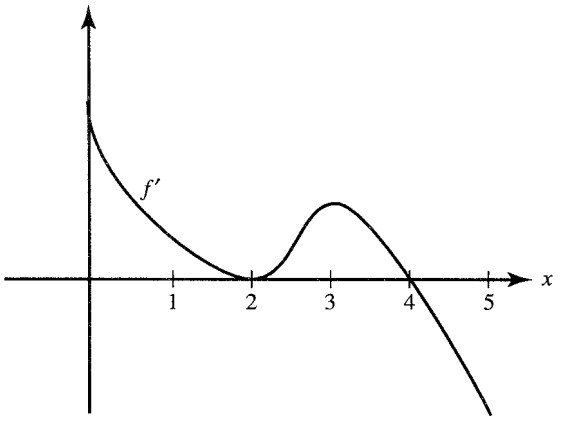
(A) −8.08 **(B)** 7.92 **(C)** 7.98 **(D)** 8.02 **(E)** 8.08

30. An object moving along a line has velocity $v(t) = t \cos t - \ln(t + 2)$, where $0 \leq t \leq 10$. How many times does the object reverse direction?

(A) none **(B)** one **(C)** two **(D)** three **(E)** four

†Beginning in May 1998, 50 minutes will be allowed for Part B.

The graph of f' shown is for Questions 31 and 32.



31. f has a local minimum at $x =$
(A) 0 only **(B)** 4 only **(C)** 0 and 4 **(D)** 0 and 5 **(E)** 0, 4, and 5

32. f has a point of inflection at $x =$
(A) 2 only **(B)** 3 only **(C)** 4 only **(D)** 2 and 3 only
(E) 2, 3, and 4

33. If c is the value defined by the Mean Value Theorem, then for $f(x) = e^x - x^2$ on $[0,1]$, $c =$
(A) -0.248 **(B)** 0.351 **(C)** 0.500 **(D)** 0.693 **(E)** 0.718

34. Find the volume of the solid generated when the region bounded by the y-axis, $y = e^x$, and $y = 2$ is rotated around the y-axis.
(A) 0.296 **(B)** 0.592 **(C)** 2.427 **(D)** 3.998 **(E)** 27.577

35. If y is a differentiable function of x, then the slope of the curve of $xy^2 - 2y + 4y^3 = 6$ at the point where $y = 1$ is
(A) $-\frac{1}{18}$ **(B)** $-\frac{1}{26}$ **(C)** $\frac{5}{18}$ **(D)** $-\frac{11}{18}$ **(E)** 2

36. The acceleration of a particle moving along a straight line is given by $a = 6t$. If, when $t = 0$, its velocity, v, is 1 and its position, s, is 3, then at any time t
(A) $s = t^3 + 3$ **(B)** $s = t^3 + 3t + 1$ **(C)** $s = t^3 + t + 3$
(D) $s = \frac{t^3}{3} + t + 3$ **(E)** $s = \frac{t^3}{3} + \frac{t^2}{2} + 3$

37. If $y = f(x^2)$ and $f'(x) = \sqrt{5x - 1}$ then $\frac{dy}{dx}$ is equal to
(A) $2x\sqrt{5x^2 - 1}$ **(B)** $\sqrt{5x - 1}$ **(C)** $2x\sqrt{5x - 1}$ **(D)** $\frac{\sqrt{5x - 1}}{2x}$
(E) none of these

38. If the area under $y = \sin x$ is equal to the area under $y = x^2$ between $x = 0$ and $x = k$, then $k =$
(A) -1.105 **(B)** 0.877 **(C)** 1.105 **(D)** 1.300 **(E)** 1.571

39. $\int \sin(x^2)\,dx =$

(A) $-\cos(x^2) + C$ **(B)** $\cos(x^2) + C$ **(C)** $-\dfrac{\cos x^2}{2x} + C$

(D) $2x \cos x^2 + C$ **(E)** none of these

40. At noon, an experimenter has 50 grams of a radioactive isotope. At noon nine days later only 45 grams remain. To the nearest day, how many days after the experiment started will there be only 20 grams?

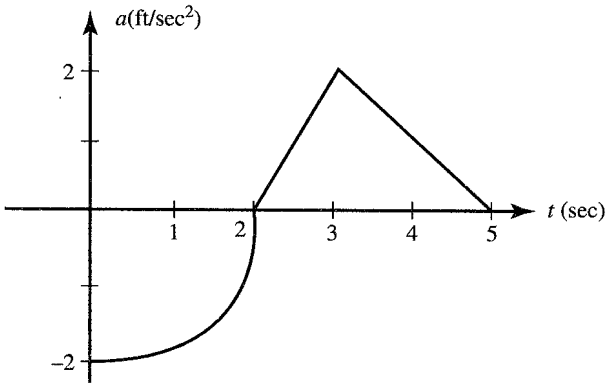
(A) 54 **(B)** 59 **(C)** 60 **(D)** 75 **(E)** 78

41. A 26-ft ladder leans against a building so that its foot moves away from the building at the rate of 3 ft/sec. When the foot of the ladder is 10 ft from the building, the top is moving down at the rate of r ft/sec, where r is

(A) $\dfrac{46}{3}$ **(B)** $\dfrac{3}{4}$ **(C)** $\dfrac{5}{4}$ **(D)** $\dfrac{5}{2}$ **(E)** $\dfrac{4}{5}$

42. If $F(x) = \displaystyle\int_1^{2x} \frac{1}{1-t^3}\,dt$, then $F'(x) =$

(A) $\dfrac{1}{1-x^3}$ **(B)** $\dfrac{1}{1-2x^3}$ **(C)** $\dfrac{2}{1-2x^3}$ **(D)** $\dfrac{1}{1-8x^3}$ **(E)** $\dfrac{2}{1-8x^3}$



43. The graph shows an object's acceleration (in ft/sec^2). It consists of a quarter-circle and two line segments. If the object was at rest at $t = 5$ sec, what was its initial velocity?

(A) -2 ft/sec **(B)** $3 - \pi$ ft/sec **(C)** 0 ft/sec **(D)** $\pi - 3$ ft/sec

(E) $\pi + 3$ ft/sec

44. Water is leaking from a tank at the rate of $R(t) = 5 \arctan\left(\dfrac{t}{5}\right)$ gallons per hour, where t is the number of hours since the leak began. How many gallons will leak out during the first day?

(A) 7 **(B)** 82 **(C)** 124 **(D)** 141 **(E)** 164

45. Find the y-intercept of the line tangent to $y = (x^3 - 4x^2 + 8)e^{\cos x^2}$ at $x = 2$.

(A) -21.032 **(B)** -2.081 **(C)** 0 **(D)** 4.161 **(E)** 21.746

Answers to AB Practice Examination 1: Section I

1. C	**10.** D	**19.** B	**28.** C	**37.** A
2. D	**11.** E	**20.** A	**29.** B	**38.** D
3. A	**12.** D	**21.** D	**30.** C	**39.** E
4. D	**13.** E	**22.** C	**31.** D	**40.** E
5. E	**14.** D	**23.** C	**32.** D	**41.** C
6. D	**15.** B	**24.** B	**33.** B	**42.** E
7. E	**16.** D	**25.** D	**34.** B	**43.** D
8. B	**17.** E	**26.** A	**35.** A	**44.** C
9. C	**18.** D	**27.** E	**36.** C	**45.** D