AB Practice Examination 4

Section I

Part A†

(See instructions, page 467. Answers begin on page 523.)

1. $\lim_{x\to 2}[x]$ (where $[x]$ is the greatest integer in x) is

(A) 1 **(B)** 2 **(C)** 3 **(D)** ∞ **(E)** nonexistent

2. $\lim_{h\to 0} \dfrac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}$ is

(A) 1 **(B)** -1 **(C)** 0 **(D)** ∞ **(E)** none of these

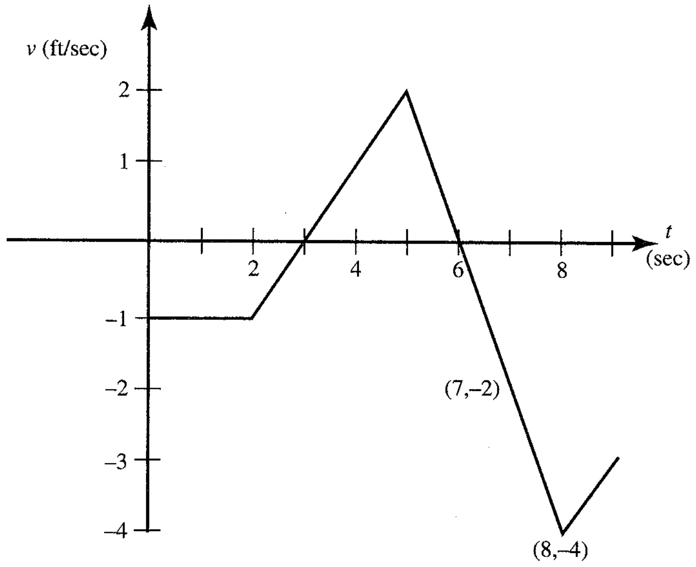
3. If $f(x) = x \ln x$, then $f'''(e)$ equals

(A) $\frac{1}{e}$ **(B)** 0 **(C)** $-\frac{1}{e^2}$ **(D)** $\frac{1}{e^2}$ **(E)** $\frac{2}{e^3}$

4. The equation of the tangent to the curve $2x^2 - y^4 = 1$ at the point $(-1, 1)$ is

(A) $y = -x$
(B) $y = 2 - x$
(C) $4y + 5x + 1 = 0$
(D) $x - 2y + 3 = 0$
(E) $x - 4y + 5 = 0$

†Beginning in May 1998, 55 minutes will be allowed for Part A.



The graph shown is for Questions 5 and 6. It shows the velocity of an object during the interval $0 \leqslant t \leqslant 9$.

5. The object attains its greatest speed at $t =$
(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

6. The object was at the origin at $t = 3$. It returned to the origin
(A) at $t = 5$ **(B)** at $t = 6$ **(C)** during $6 < t < 7$
(D) at $t = 7$ **(E)** during $7 < t < 8$

7. A relative maximum value of the function $y = \dfrac{\ln x}{x}$ is

(A) 1 **(B)** e **(C)** $\dfrac{2}{e}$ **(D)** $\dfrac{1}{e}$ **(E)** none of these

8. If a particle moves on a line according to the law $s = t^5 + 2t^3$, then the number of times it reverses direction is
(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

9. A rectangular pigpen is to be built against a wall so that only three sides will require fencing. If p feet of fencing are to be used, the area of the largest possible pen is
(A) $\dfrac{p^2}{2}$ **(B)** $\dfrac{p^2}{4}$ **(C)** $\dfrac{p^2}{8}$ **(D)** $\dfrac{p^2}{9}$ **(E)** $\dfrac{p^2}{16}$

10. Let $F(x) = \displaystyle\int_5^x \frac{dt}{1 - t^2}$. Which is true?
I. The domain of F is $x \neq \pm 1$. **II.** $F(2) > 0$. **III.** F is concave upward.
(A) none **(B)** I only **(C)** II only **(D)** III only
(E) II and III only

11. As the tides change, the water-level in a bay varies sinusoidally. At high tide today at 8 A.M., the water-level was 15 feet; at low tide, 6 hours later at 2 P.M., it was 3 feet. How fast, in ft per hr, was the water-level dropping at noon today?

(A) 3 **(B)** $\frac{\pi\sqrt{3}}{2}$ **(C)** $3\sqrt{3}$ **(D)** $\pi\sqrt{3}$ **(E)** $6\sqrt{3}$

12. A smooth curve with equation $y = f(x)$ is such that its slope at each x equals x^2. If the curve goes through the point $(-1, 2)$, then its equation is

(A) $y = \frac{x^3}{3} + 7$ **(B)** $x^3 - 3y + 7 = 0$

(C) $y = x^3 + 3$ **(D)** $y - 3x^3 - 5 = 0$ **(E)** none of these

13. $\int \frac{e^u}{4 + e^{2u}}\, du$ is equal to

(A) $\ln(4 + e^{2u}) + C$ **(B)** $\frac{1}{2}\ln|4 + e^{2u}| + C$

(C) $\frac{1}{2}\tan^{-1}\frac{e^u}{2} + C$ **(D)** $\tan^{-1}\frac{e^u}{2} + C$

(E) $\frac{1}{2}\tan^{-1}\frac{e^{2u}}{2} + C$

14. Given $f(x) = \log_{10} x$ and $\log_{10}(102) \simeq 2.0086$, which is closest to $f'(100)$?
(A) 0.0043 **(B)** 0.0086 **(C)** 0.01 **(D)** 1.0043 **(E)** 2

15. If $G(2) = 5$ and $G'(x) = \frac{10x}{9 - x^2}$, then an estimate of $G(2.2)$ using local linearization is approximately

(A) 5.4 **(B)** 5.5 **(C)** 5.8 **(D)** 8.8 **(E)** 13.8

16. The area bounded by the parabola $y = x^2$ and the lines $y = 1$ and $y = 9$ equals

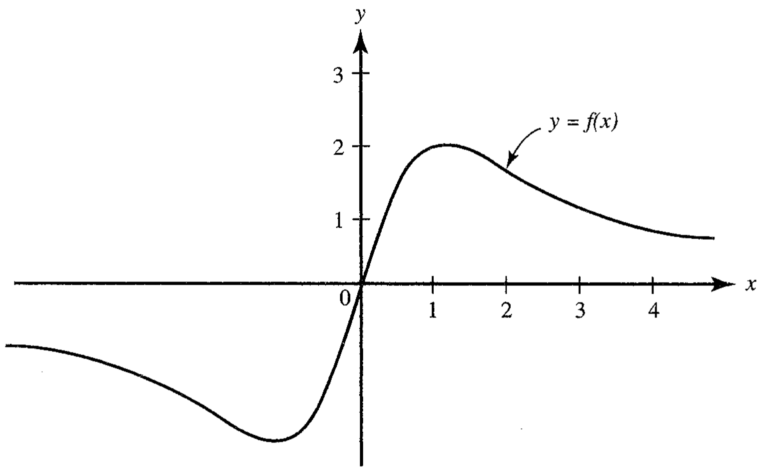
(A) 8 **(B)** $\frac{84}{3}$ **(C)** $\frac{64}{3}\sqrt{2}$ **(D)** 32 **(E)** $\frac{104}{3}$

17. Suppose $f(x) = \frac{x^2 + x}{x}$ if $x \neq 0$ and $f(0) = 1$. Which of the following statements are true of f?

I. f is defined at $x = 0$.
II. $\lim_{x \to 0} f(x)$ exists.
III. f is continuous at $x = 0$.

(A) I only **(B)** II only **(C)** I and II only
(D) None of the statements is true. **(E)** All are true.

18. Which function could have the graph shown below?



(A) $y = \dfrac{x}{x^2 + 1}$ **(B)** $y = \dfrac{4x}{x^2 + 1}$ **(C)** $y = \dfrac{2x}{x^2 - 1}$ **(D)** $y = \dfrac{x^2 + 3}{x^2 + 1}$

(E) $y = \dfrac{4x}{x + 1}$

19. Suppose the function f is both increasing and concave up on $a \leqslant x \leqslant b$. Then, using the same number of subdivisions, and with L, R, M, and T denoting respectively Left, Right, Midpoint, and Trapezoid sums, it follows that

(A) $R \leqslant T \leqslant M \leqslant L$ **(B)** $L \leqslant T \leqslant M \leqslant R$

(C) $R \leqslant M \leqslant T \leqslant L$ **(D)** $L \leqslant M \leqslant T \leqslant R$

(E) none of these

20. $\lim\limits_{x \to 3} \dfrac{x + 3}{x^2 - 9}$ is

(A) $+\infty$ **(B)** 0 **(C)** $\dfrac{1}{6}$ **(D)** $-\infty$ **(E)** nonexistent

21. The only function that does not satisfy the Mean Value Theorem on the interval specified is

(A) $f(x) = x^2 - 2x$ on $[-3,1]$. **(B)** $f(x) = \dfrac{1}{x}$ on $[1,3]$.

(C) $f(x) = \dfrac{x^3}{3} - \dfrac{x^2}{2} + x$ on $[-1, 2]$. **(D)** $f(x) = x + \dfrac{1}{x}$ on $[-1, 1]$.

(E) $f(x) = x^{2/3}$ on $\left[\dfrac{1}{2}, \dfrac{3}{2}\right]$.

22. Suppose $f'(x) = x(x - 2)^2(x + 3)^2$. Which of the following is (are) true?

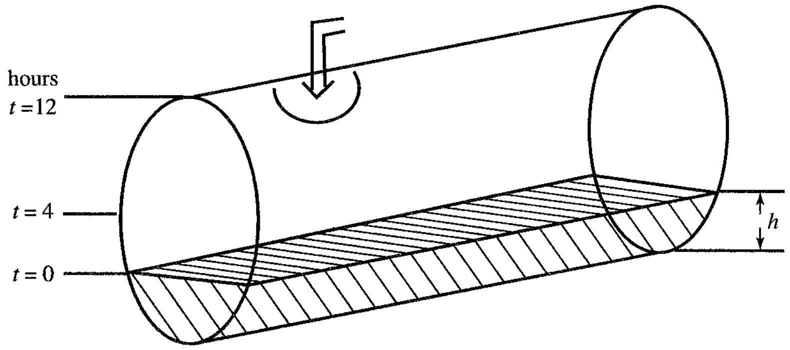
I. f has a local maximum at $x = -3$.

II. f has a local minimum at $x = 0$.

III. f has neither a local maximum nor a local minimum at $x = 2$.

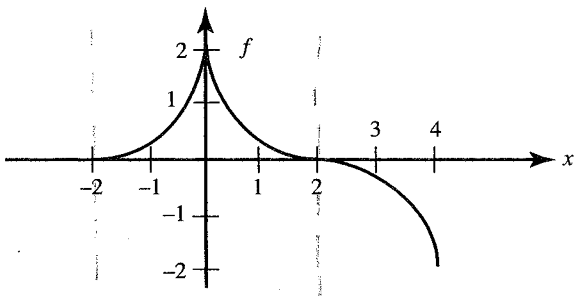
(A) I only **(B)** II only **(C)** III only **(D)** I and II only

(E) I, II, and III



23. A cylindrical tank is partially full of water at time $t = 0$, when more water begins flowing in at a constant rate. The tank becomes half full when $t = 4$, and is completely full when $t = 12$. Let h represent the height of the water at time t. During which interval is $\frac{dh}{dt}$ increasing?

(A) never **(B)** $0 < t < 4$ **(C)** $0 < t < 8$ **(D)** $0 < t < 12$
(E) $4 < t < 12$



24. The graph of function f consists of 3 quarter-circles.

Which of the following is equivalent to $\int_0^2 f(x)\,dx$?

I. $\frac{1}{2}\int_{-2}^{2} f(x)\,dx$ **II.** $\int_4^2 f(x)\,dx$ **III.** $\frac{1}{2}\int_0^4 f(x)\,dx$

(A) I only **(B)** II only **(C)** III only **(D)** I and II only
(E) all of these

25. The base of a solid is the first-quadrant region bounded by $y = \sqrt[4]{4 - 2x}$, and each cross-section perpendicular to the x-axis is a semicircle with a diameter in the xy-plane. The volume of the solid is

(A) $\frac{\pi}{2}\int_0^2 \sqrt{4 - 2x}\,dx$ **(B)** $\frac{\pi}{8}\int_0^2 \sqrt{4 - 2x}\,dx$ **(C)** $\frac{\pi}{8}\int_{-2}^2 \sqrt{4 - 2x}\,dx$

(D) $\frac{\pi}{4}\int_0^{\sqrt{2}} (4 - y^4)^2\,dy$ **(E)** $\frac{\pi}{8}\int_0^{\sqrt[4]{4}} (4 - y^4)^2\,dy$

26. The average value of $f(x) = 3 + |x|$ on the interval $[-2, 4]$ is

(A) $2\frac{2}{3}$ **(B)** $3\frac{1}{3}$ **(C)** $4\frac{2}{3}$ **(D)** $5\frac{1}{3}$ **(E)** 6

27. $\lim_{x\to\infty} \dfrac{3 + x - 2x^2}{4x^2 + 9}$ is

(A) $-\dfrac{1}{2}$ **(B)** $\dfrac{1}{2}$ **(C)** 1 **(D)** 3 **(E)** nonexistent

28. The area of the region in the xy-plane bounded by the curves $y = e^x$, $y = e^{-x}$, and $x = 1$ is equal to

(A) $e + \dfrac{1}{e} - 2$ **(B)** $e - \dfrac{1}{e}$ **(C)** $e + \dfrac{1}{e}$ **(D)** $2e - 2$

(E) none of the these

Part B†

(See instructions, page 471)

29. $f(x) = \int_0^{x^2+2} \sqrt{1 + \cos t}\, dt$. Then $f'(x) =$

(A) $2x\sqrt{1 + \cos (x^2 + 2)}$ **(B)** $2x\sqrt{1 - \sin x}$ **(C)** $(x^2 + 2)\sqrt{1 - \sin x}$

(D) $\sqrt{1 + \cos (x^2 + 2)}$ **(E)** $\sqrt{[1 + \cos (x^2 + 2)] \cdot 2x}$

30. If $y = \ln \dfrac{x}{\sqrt{x^2 + 1}}$, then $\dfrac{dy}{dx}$ is

(A) $\dfrac{1}{x^2 + 1}$ **(B)** $\dfrac{1}{x(x^2 + 1)}$ **(C)** $\dfrac{2x^2 + 1}{x(x^2 + 1)}$ **(D)** $\dfrac{1}{x\sqrt{x^2 + 1}}$

(E) $\dfrac{1 - x^2}{x(x^2 + 1)}$

31. A particle moves on a line according to the law $s = f(t)$ so that its velocity $v = ks$, where k is a nonzero constant. Its acceleration is

(A) k^2v **(B)** k^2s **(C)** k **(D)** 0 **(E)** none of these

32. A cup of coffee placed on a table cools at a rate of $\dfrac{dH}{dt} = -0.05(H - 70)$ degrees per minute, where H represents its temperature and t is time in minutes. If the coffee was at 120°F initially, what will its temperature be 10 minutes later?

(A) 73°F **(B)** 95°F **(C)** 100°F **(D)** 118°F **(E)** 143°F

33. An investment of \$4000 grows at the rate of $320e^{0.08t}$ dollars per year after t years. Its value after 10 years is approximately

(A) \$4902 **(B)** \$8902 **(C)** \$7122 **(D)** \$12,902

(E) none of these

34. If $f(x) = (1 + e^x)$ then the domain of $f^{-1}(x)$ is

(A) $(-\infty,\infty)$ **(B)** $(0,\infty)$ **(C)** $(1,\infty)$ **(D)** $\{x \mid x \geq 1\}$

(E) $\{x \mid x \geq 2\}$

†Beginning in May 1998, 50 minutes will be allowed for Part B.

35. The intervals on which the function $f(x) = x^4 - 4x^3 + 4x^2 + 6$ increases are
(A) $x < 0$ and $1 < x < 2$ **(B)** only $x > 2$ **(C)** $0 < x < 1$ and $x > 2$
(D) only $0 < x < 1$ **(E)** only $1 < x < 2$

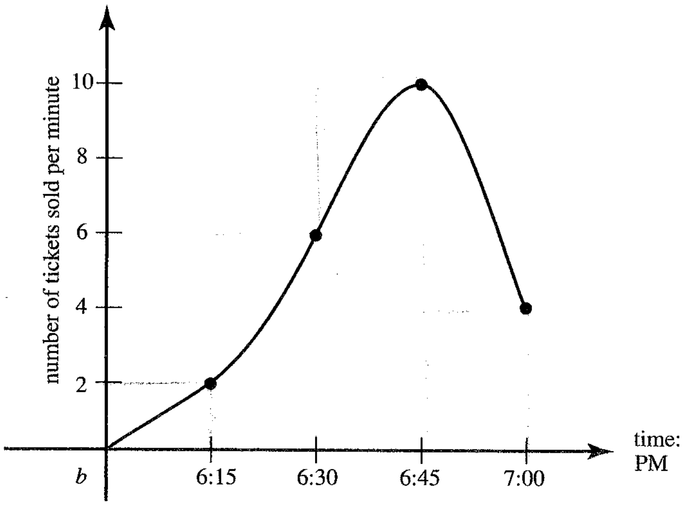
36. $\int \frac{\cos x}{4 + 2\sin x}\,dx$ equals
(A) $\sqrt{4 + 2\sin x} + C$ **(B)** $-\frac{1}{2(4 + \sin x)} + C$
(C) $\ln\sqrt{4 + 2\sin x} + C$ **(D)** $2\ln|4 + 2\sin x| + C$
(E) $\frac{1}{4}\sin x - \frac{1}{2}\csc^2 x + C$

37. The region bounded by the y-axis, $y = e^x$, and $y = k$ is rotated around the x-axis to form a solid with volume π. Then $k =$
(A) 1.732 **(B)** 1.895 **(C)** 1.911 **(D)** 2.048 **(E)** 2.149

38. If we replace $\sqrt{x - 2}$ by u, then $\int_3^6 \frac{\sqrt{x - 2}}{x}\,dx$ is equivalent to
(A) $\int_1^2 \frac{u\,du}{u^2 + 2}$ **(B)** $2\int_1^2 \frac{u^2\,du}{u^2 + 2}$ **(C)** $\int_3^6 \frac{2u^2}{u^2 + 2}\,du$ **(D)** $\int_3^6 \frac{u\,du}{u^2 + 2}$
(E) $\frac{1}{2}\int_1^2 \frac{u^2}{u^2 + 2}\,du$

39. The total area enclosed by the curves $y = 4x - x^3$ and $y = 4 - x^2$ is represented by
(A) $\int_{-2}^{2} (4x - x^3 + x^2 - 4)\,dx$
(B) $\int_{-2}^{2} (4 - x^2 - 4x + x^3)\,dx$
(C) $\int_{-2}^{1} (4x - x^3 + x^2 - 4)\,dx + \int_{1}^{2} (4 - x^2 - 4x + x^3)\,dx$
(D) $\int_{-2}^{1} (4 - x^2 - 4x + x^3)\,dx + \int_{1}^{2} (4x - x^3 + x^2 - 4)\,dx$
(E) $\int_{-2}^{1} (4x - x^3 + x^2 - 4)\,dx$

40. How many points of inflection does the function f have on the interval $0 \leqslant x \leqslant 6$ if $f''(x) = 2 - 3\sqrt{x}\cos^3 x$?
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

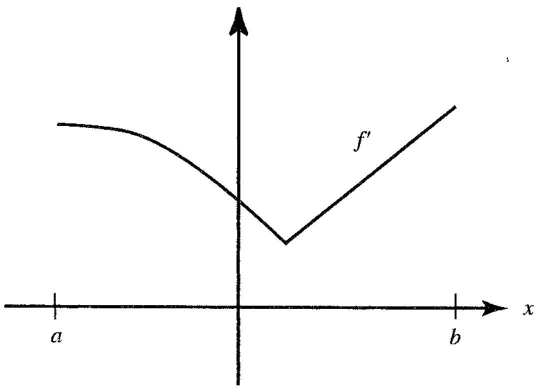


41. The graph shows the rate at which tickets were sold at a movie theater during the last hour before showtime. Using the right-rectangle method, an estimate of the size of the audience is
(A) 230 **(B)** 300 **(C)** 330 **(D)** 375 **(E)** 420

42. At what point of intersection of $f(x) = 4^{\sin x}$ and $g(x) = \ln(x^2)$ do their derivatives have the same sign?
(A) -5.2 **(B)** -4.0 **(C)** -1.2 **(D)** 2.6 **(E)** 7.8

43. Let C be the arc of $y = 8 - \frac{1}{2}x^2$ which lies above the x-axis. Find the longest line segment both of whose endpoints lie on C.
(A) 7.7 **(B)** 8 **(C)** 8.9 **(D)** 9.1 **(E)** 12.5

44. The graph of f' appears below. Which statements about f must be true for $a < x < b$?



I. f is increasing **II.** f is continuous **III.** f is differentiable
(A) I only **(B)** II only **(C)** I and II only **(D)** I and III only
(E) all three

45. After a bomb explodes, pieces can be found scattered around the center of the blast. The density of bomb fragments lying x meters from ground zero is given by $N(x) = \frac{2x}{1 + x^{3/2}}$ fragments per square meter. How many fragments will be found within 20 meters of the point where the bomb exploded?

(A) 13 **(B)** 278 **(C)** 556 **(D)** 712 **(E)** 4383

Answers to AB Practice Examination 4: Section I

1.	E	**10.**	E	**19.**	D	**28.**	A	**37.**	B
2.	C	**11.**	B	**20.**	E	**29.**	A	**38.**	B
3.	C	**12.**	B	**21.**	D	**30.**	B	**39.**	D
4.	A	**13.**	C	**22.**	E	**31.**	B	**40.**	A
5.	E	**14.**	A	**23.**	E	**32.**	C	**41.**	C
6.	E	**15.**	C	**24.**	D	**33.**	B	**42.**	B
7.	D	**16.**	E	**25.**	B	**34.**	C	**43.**	D
8.	E	**17.**	E	**26.**	C	**35.**	C	**44.**	E
9.	C	**18.**	B	**27.**	A	**36.**	C	**45.**	D

Part A

1. E. Here, $\lim_{x\to 2^-} [x] = 1$, while $\lim_{x\to 2^+} [x] = 2$.

2. C. The given limit equals $f'\left(\frac{\pi}{2}\right)$, where $f(x) = \sin x$.

3. C. Since $f(x) = x \ln x$,

$$f'(x) = 1 + \ln x, \quad f''(x) = \frac{1}{x}, \quad \text{and} \quad f'''(x) = -\frac{1}{x^2}.$$

4. A. Differentiate implicitly to get $4x - 4y^3 \frac{dy}{dx} = 0$. Substitute $(-1, 1)$ to find $\frac{dy}{dx}$, the slope, at this point, and write the equation of the tangent: $y - 1 = -1(x + 1)$.

5. E. Speed is the magnitude of velocity: $|v(8)| = 4$.

6. E. For $3 < t < 6$ the object travels to the right $\frac{1}{2}(3)(2) = 3$ units. At $t = 7$ it has returned 1 unit to the left; by $t = 8$, 4 units to the left.

7. D. Here $y' = \frac{1 - \ln x}{x^2}$, which is zero for $x = e$. Since the signs change from positive to negative as x increases through e, this critical value yields a relative maximum. Note that $f(e) = \frac{1}{e}$.

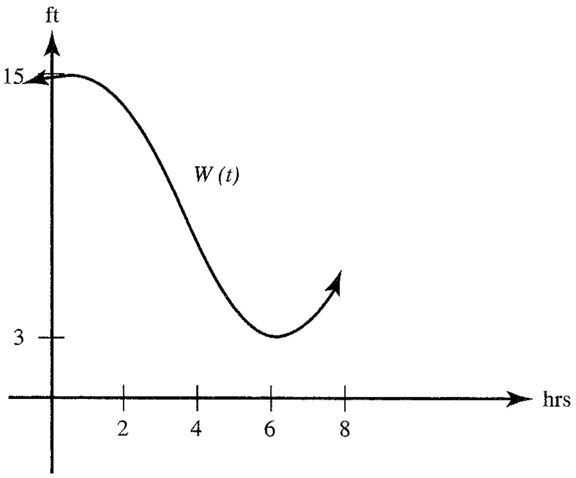
8. E. Since $v = \frac{ds}{dt} = 5t^4 + 6t^2$ never changes signs, there are no reversals in motion along the line.

9. C. Letting y be the length parallel to the wall and x the other dimension of the rectangle, we have

$$p = 2x + y \text{ and } A = xy = x(p - 2x).$$

For $x = \frac{p}{4}$ we have $\frac{dA}{dx} = 0$, which yields $y = \frac{p}{2}$. Note that $\frac{d^2A}{dx^2} < 0$.

10. E. Since $f(x) = F'(x) = \frac{1}{1 - x^2}$, f is discontinuous at $x = 1$; the domain of F is therefore $x > 1$. On [2, 5] $f(x) < 0$, so $\int_5^2 f > 0$. $F''(x) = f'(x) = \frac{2x}{(1 - x^2)^2}$, which is positive for $x > 1$.



11. B. $W(t)$, the water-level at time t, is a cosine function with amplitude 6 feet and period 12 hours:

$$W(t) = 6 \cos\left(\frac{\pi}{6} t\right) + 9 \text{ ft}$$

$$W'(t) = -\pi \sin\left(\frac{\pi}{6} t\right) \text{ ft per hr}$$

Find $W'(4)$.

12. B. We solve the differential equation $\frac{dy}{dx} = x^2$, getting $y = \frac{x^3}{3} + C$. Use $x = -1$, $y = 2$ to determine C.

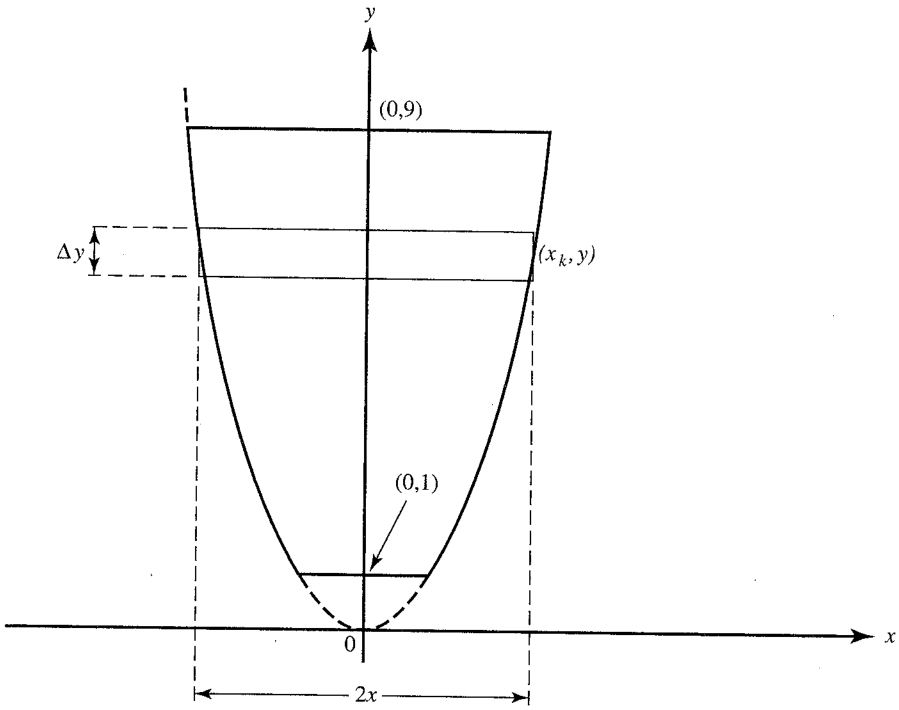
13. C. Note that the given integral is of the type

$$\int \frac{dv}{a^2 + v^2} = \frac{1}{a} \tan^{-1} \frac{v}{a} + C.$$

14. A. $f'(100) \simeq \dfrac{f(102) - f(100)}{102 - 100} = \dfrac{2.0086 - 2}{2}$.

15. C. $G'(2) = 4$, so $G(x) \simeq 4(x - 2) + 5$.

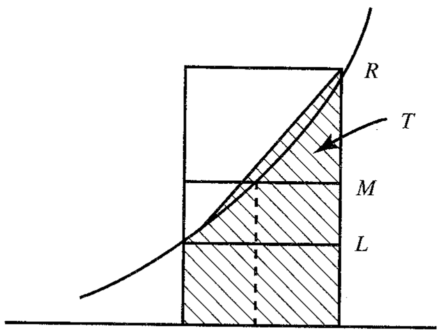
16. E. $A = 2\int_1^9 x\,dy = 2\int_1^9 \sqrt{y}\,dy = \dfrac{104}{3}$.



17. E. Note that $\lim_{x\to 0} f(x) = f(0) = 1$.

18. B. Note that (0, 0) is on the graph, as are (1, 2) and (−1, −2). So only (B) and (E) are possible. Since $\lim_{x\to\infty} y = \lim_{x\to -\infty} y = 0$, only (B) is correct.

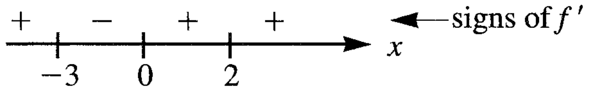
19. D. See miscellaneous example 33, page 448.



20. E. $\dfrac{x+3}{x^2-9} = \dfrac{x+3}{(x+3)(x-3)} = \dfrac{1}{x-3}$; $\lim_{x\to 3^+} \dfrac{1}{x-3} = +\infty$; $\lim_{x\to 3^-} \dfrac{1}{x-3} = -\infty$.

21. D. In (D), $f(x)$ is not defined at $x = 0$. Verify that each of the other functions satisfies both conditions of the Mean Value Theorem.

22. E. The signs within the intervals bounded by the critical points are given by



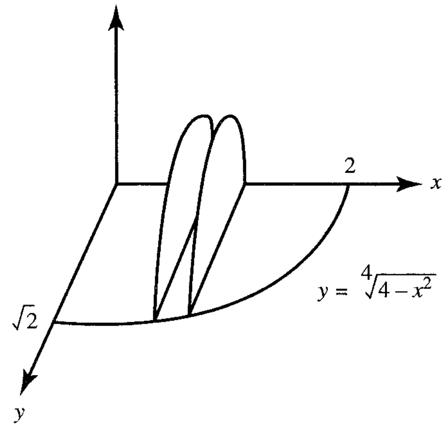
f has a local maximum at -3 and a local minimum at 0.

23. E. $\frac{dh}{dt}$ will increase (that is, the height of the water will rise more rapidly) as the area of the cross section diminishes—above the half-full level.

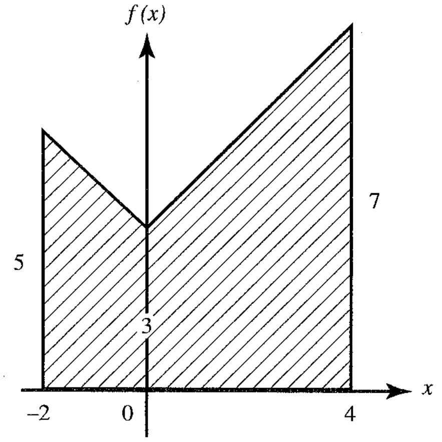
24. D. $\int_{-2}^{0} f = \int_{0}^{2} f = -\int_{2}^{4} f$, but $\int_{0}^{4} f = 0$.

25. B. $\Delta V = \frac{1}{2}\pi r^2 \Delta x$ where $y = 2r$.

$$V = \frac{\pi}{2}\int_0^2 \left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8}\int_0^2 \sqrt{4 - 2x}\, dx$$



26. C. $$\frac{\int_{-2}^{4} f}{6} = \frac{\frac{5+3}{2}\cdot 2 + \frac{3+7}{2}\cdot 4}{6} = \frac{28}{6}.$$

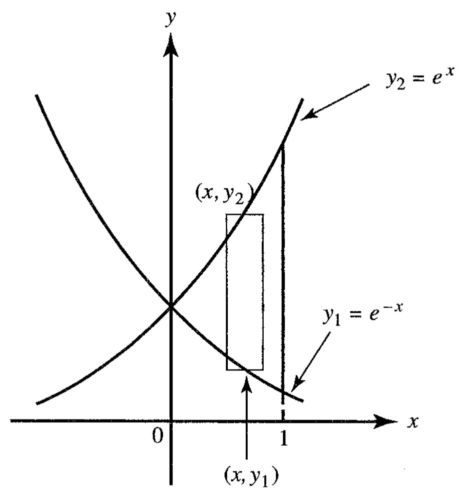


27. A. Since the degrees of numerator and denominator are the same, the limit as $x \to \infty$ is the ratio of the coefficients of the terms of highest degree: $\frac{-2}{4}$.

28. A. $\Delta A = (y_2 - y_1)\,\Delta x$;

$$A = \int_0^1 (e^x - e^{-x})\,dx$$

$$= (e^x + e^{-x})\Big|_0^1 = e + \frac{1}{e} - 2.$$



Part B

29. A. Let $u = x^2 + 2$. Then

$$\frac{d}{du}\int_0^u \sqrt{1 + \cos t}\;dt = \sqrt{1 + \cos u}$$

and

$$\frac{d}{dx}\int_0^u \sqrt{1 + \cos t}\;dt = \sqrt{1 + \cos u}\,\frac{du}{dx} = \sqrt{1 + \cos (x^2 + 2)}\cdot(2x).$$

30. B. Since $\ln \frac{x}{\sqrt{x^2+1}} = \ln x - \frac{1}{2} \ln (x^2+1)$, then

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{2} \cdot \frac{2x}{x^2+1} = \frac{1}{x(x^2+1)}.$$

31. B. Since $v = ks = \frac{ds}{dt}$, then $a = \frac{d^2s}{dt^2} = k\frac{ds}{dt} = kv = k^2 s$.

32. C. $\frac{dH}{H-70} = -0.05\,dt.\quad \ln|H-70| = -0.05t + C$

$$H - 70 = ce^{-0.05t}$$

$$H(x) = 70 + ce^{-0.05t}$$

The initial condition $H(0) = 120$ shows $c = 50$. Evaluate $H(10)$.

33. B. Let P be the amount after t years. We are given that $\frac{dP}{dt} = 320e^{0.08t}$. The solution of this differential equation is $P = 4000e^{0.08t} + C$, where $P(0) = 4000$ yields $C = 0$. The answer is $4000e^{(0.08)\cdot 10}$.

34. C. The inverse of $y = 1 + e^x$ is $x = 1 + e^y$ or $y = \ln(x-1)$; $(x-1)$ must be positive.

35. C. $f'(x) = 4x^3 - 12x^2 + 8x = 4x(x-1)(x-2)$. To determine the signs of $f'(x)$, inspect the sign at any point in each of the intervals $x < 0$, $0 < x < 1$, $1 < x < 2$, and $x > 2$. The function increases whenever $f'(x) > 0$.

36. C. The integral is equivalent to $\frac{1}{2}\int \frac{du}{u}$, where $u = 4 + 2\sin x$.

37. B. Using washers,

$$\Delta V = \pi(R^2 - r^2)\Delta x$$

$$= \pi(k^2 - (e^x)^2)\,\Delta x$$

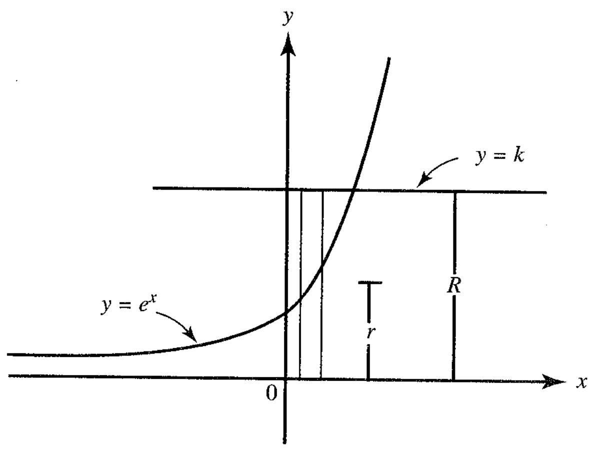
So

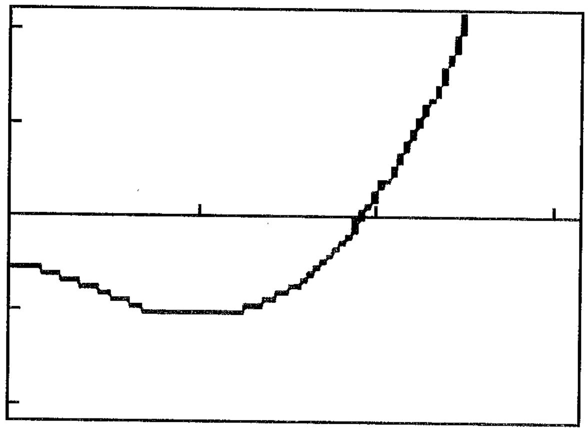
$$V = \pi\int_0^{\ln k} (k^2 - e^{2x})\,dx = \pi$$

$$\left(k^2 x - \frac{e^{2x}}{2}\right)\Bigg|_0^{\ln k} = 1$$

$$\left(k^2 \ln k - \frac{e^{2\ln k}}{2}\right) - \left(0 - \frac{1}{2}\right) = 1$$

$$k^2 \ln k - \frac{k^2}{2} - \frac{1}{2} = 0$$





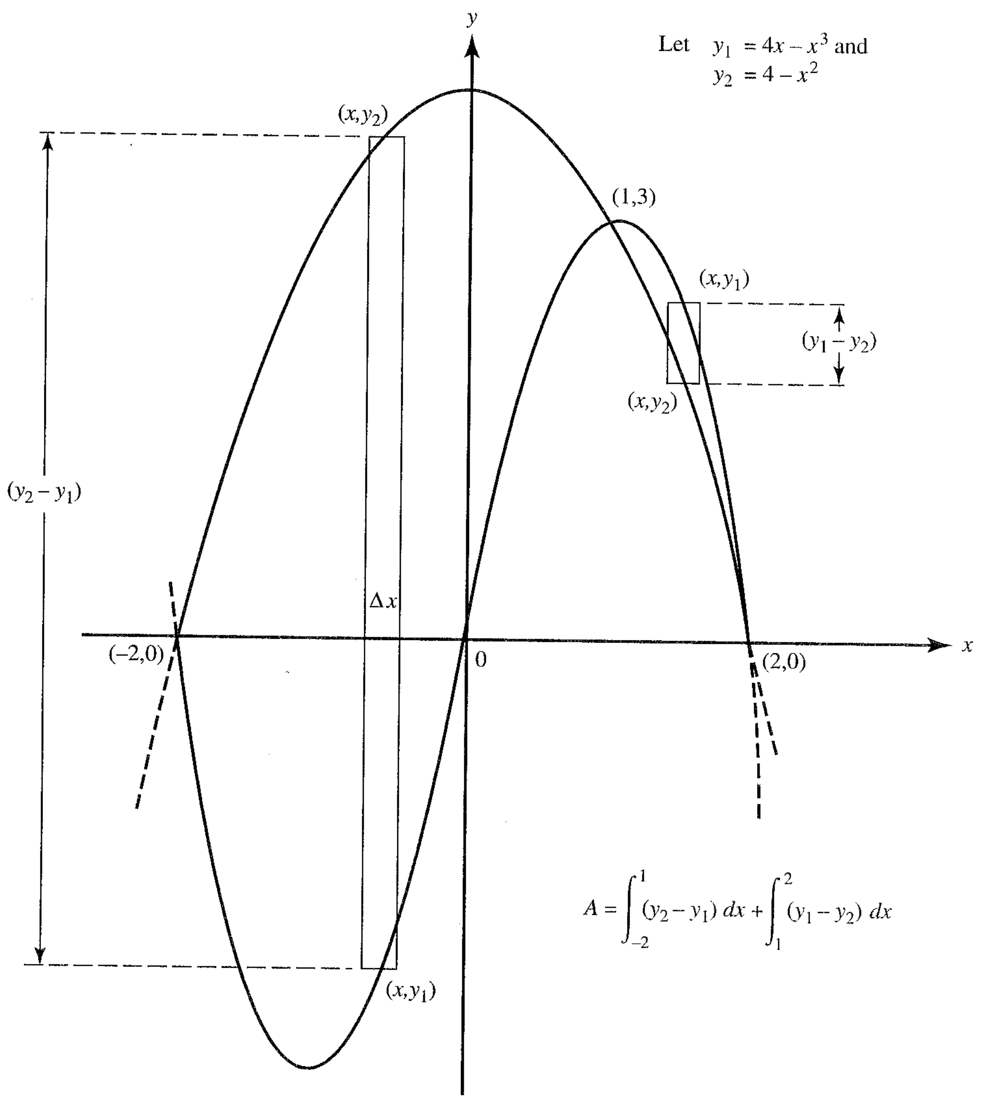
Set Y_1 equal to $X^2 \ln X - (X^2/2) - (1/2)$ and graph Y_1 in `[0,3]×[-2,2]`. There is a root near X = 2. To find k,

$$\text{solve } (Y_1, X, 2)$$

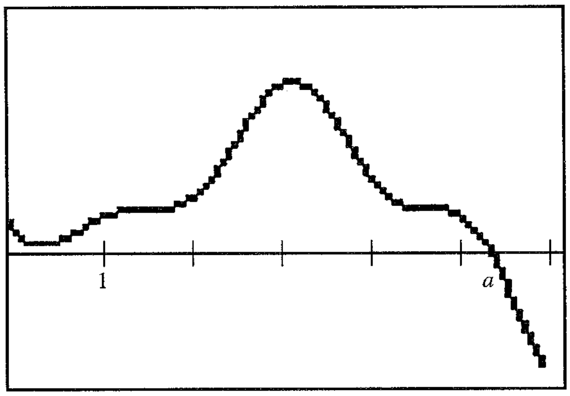
This yields $k = 1.895$. Or we can use the [root] option on the graph.

38. B. If $u = \sqrt{x - 2}$, then $u^2 = x - 2$, $x = u^2 + 2$, $dx = 2u\,du$. When $x = 3$, $u = 1$; when $x = 6$, $u = 2$.

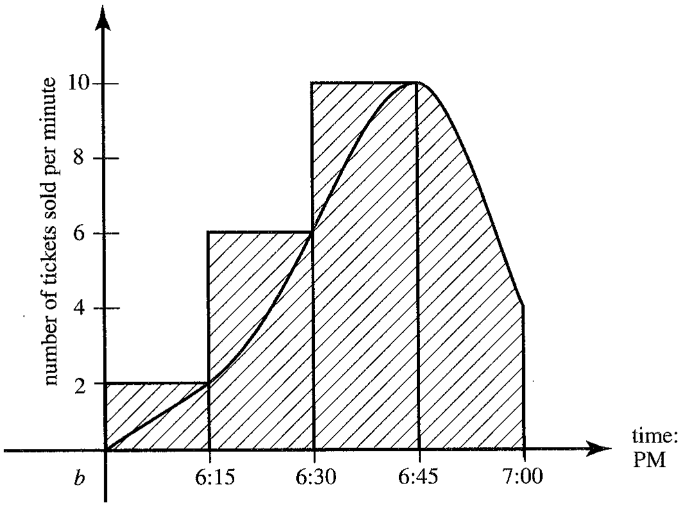
39. D.



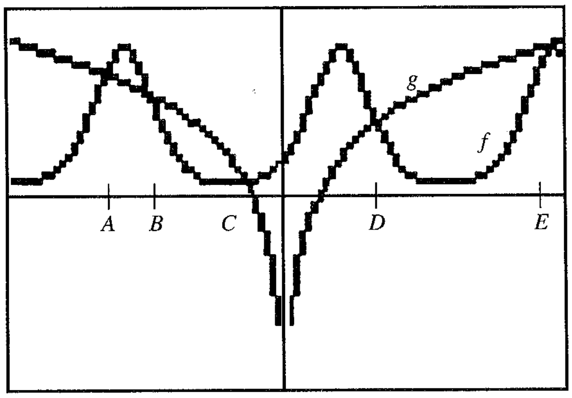
40. A. Graph f'' in `[0,6]` × `[-5,10]`. The sign of f'' changes only at $x = a$.



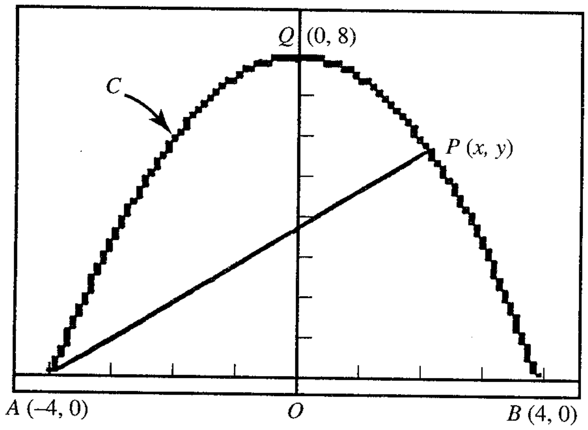
41. C. The first rectangle shows 2 tickets sold per minute for 15 minutes, or 30 tickets. Similarly, the total is 2(15) + 6(15) + 10(15) + 4(15).



42. B. Graph both functions in `[-8,8]` × `[-5,5]`. At point of intersection B, both are decreasing. Tracing reveals $x \simeq -4$ at B. If you zoom in on the curves at $x = E$, you will note that they do not actually intersect there.



Note: Scales are unequal on x- and y-axes in the figure, so that OQ appears shorter than AB, although they are equal. But the lengths given below are correct.



43. D. $AB = 8$, $AQ = \sqrt{4^2 + 8^2} = 8.944$. Could some segment AP be longer?

$AP = \sqrt{(x + 4)^2 + (y - 0)^2}$. Graph $Y_1 = \sqrt{\left((X + 4)^2 + \left(8 - \frac{1}{2}X^2\right)^2\right)}$

and note that its maximum occurs at (0.586, 9.073).

44. E. $f'(x) > 0$; the curve shows that f' is defined for all $a < x < b$, so f is differentiable and therefore continuous.

45. D. Consider the blast area as a set of concentric rings; one is shown in the figure. The area of this ring, which represents the region x meters from the center of the blast, may be approximated by the area of the rectangle shown. Since the number of particles in the ring is the area times the density, $\Delta P = 2\pi x \cdot \Delta x \cdot N(x)$. To find the total number of fragments within 20 meters of the point of the explosion, we integrate: $2\pi \int_0^{20} x \frac{2x}{1 + x^{3/2}}\, dx \simeq 711.575$.