Answers to AB Practice Examination 3: Section I

1.	E	10.	B	19.	B	28.	A	37.	B
2.	A	11.	C	20.	C	29.	B	38.	E
3.	E	12.	A	21.	B	30.	C	39.	E
4.	D	13.	B	22.	B	31.	E	40.	E
5.	B	14.	D	23.	E	32.	D	41.	D
6.	D	15.	B	24.	E	33.	D	42.	D
7.	B	16.	E	25.	B	34.	C	43.	C
8.	B	17.	C	26.	A	35.	C	44.	E
9.	B	18.	D	27.	E	36.	D	45.	B

Part A

1. E. $\frac{x^2-2}{4-x^2} \to \pm\infty$ as $x \to 2$.

2. A. Divide both numerator and denominator by $\sqrt{x}$.

3. E. Since $e^{\ln u} = u$, $y = 1$.

4. D. $f(0) = 3$, and $f'(x) = \frac{1}{2}(9 + \sin 2x)^{-1/2} \cdot (2 \cos 2x)$. So, $f'(0) = \frac{1}{3}$; $y \approx \frac{1}{3}x + 3$.

5. B. $\int_0^1 \frac{60}{1+t^2}\,dt = 60 \arctan 1 = 60 \cdot \frac{\pi}{4}$.

6. D. Here $y' = 3 \sin^2(1-2x) \cos(1-2x) \cdot (-2)$.

7. B. $\frac{d}{dx}(x^2 e^{x^{-1}}) = x^2 e^{x^{-1}}\left(-\frac{1}{x^2}\right) + 2xe^{x^{-1}}$.

8. B. Let s be the distance from the origin: then

$$s = \sqrt{x^2+y^2} \quad\text{and}\quad \frac{ds}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}.$$

Since

$$\frac{dy}{dt} = 2x\frac{dx}{dt} \quad\text{and}\quad \frac{dx}{dt} = \frac{3}{2},$$

$\frac{dy}{dt} = 3x$. Substituting yields $\frac{ds}{dt} = \frac{3\sqrt{5}}{2}$.

9. B. For $f(x) = \sqrt{x}$, this limit represents $f'(25)$.

10. B. $V = \int_0^1 y^2\,dy = \int_0^1 \sqrt{1-x^2}\,dx$, which represents the area of a quadrant of the circle $x^2 + y^2 = 1$.

11. C. The integral is equivalent to $-\frac{1}{2}\int (9 - x^2)^{-1/2}(-2x\,dx)$. Use (3) on page 46 with $u = 9 - x^2$ and $n = -\frac{1}{2}$.

12. A. The integral is rewritten as

$$\frac{1}{2}\int \frac{y^2 - 2y + 1}{y}\,dy,$$

$$= \frac{1}{2}\int \left(y - 2 + \frac{1}{y}\right) dy,$$

$$= \frac{1}{2}\left(\frac{y^2}{2} - 2y + \ln|y|\right) + C.$$

13. B. $\displaystyle\int_{\pi/6}^{\pi/2} \cot x\,dx = \ln \sin x \Big|_{\pi/6}^{\pi/2} = 0 - \ln\frac{1}{2}.$

14. D. Note:

0 1 2 3

f' { |<----------positive---------->|<--negative-->|
|<increasing>|<--------decreasing--------->|

f { |<---------increasing-------->|<decreasing>|
|<concave up>|<------concave down------>|

15. B. The winning times are positive, decreasing, and concave upward.

16. E. $G(x) = H(x) + \int_0^2 f(t)\,dt.$

17. C. $f'(x) = 0$ for $x = 1$ and $f''(1) > 0$.

18. D. $a = \frac{dv}{dt} = -t^2$ yields $v = -\frac{t^3}{3} + C_1$, and

$$s = -\frac{t^4}{12} + C_1 t + C_2.$$

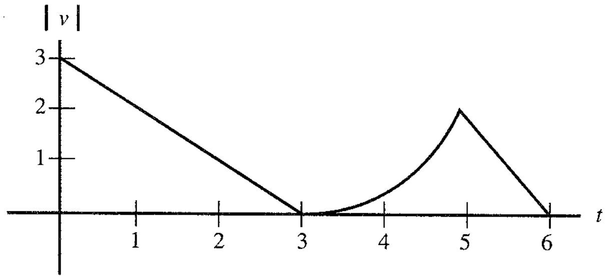
Since $s(0) = 0$, $C_2 = 0$; and since $s(1) = 3$, $C_1 = \frac{37}{12}$. Thus $v(0) = \frac{37}{12}$.

19. B. Note that

$$\lim_{x\to\infty} xe^x = \infty,\ \lim_{x\to\infty} \frac{e^x}{x} = \infty,\ \lim_{x\to-\infty} \frac{x}{x^2 + 1} = 0, \text{ and } \frac{x^2}{x^3 + 1} \geqq 0 \text{ for } x > -1.$$

20. C. v is not differentiable at $t = 3$ or $t = 5$.

21. B. Speed is the magnitude of velocity; its graph is shown below.



22. B. $\frac{v(5) - v(0)}{5 - 0} = \frac{-2 - 3}{5}$.

23. E. The curve has vertical asymptotes at $x = 2$ and $x = -2$ and a horizontal asymptote at $y = -2$.

24. E. The function is not defined at $x = -2$; $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$. Defining $f(-2) = 4$ will make f continuous at $x = -2$, but f will still be discontinuous at $x = 1$.

25. B. $(f^{-1})'(y_0)$ denotes the derivative of the inverse of $f(x)$ when $y = y_0$. Setting $2 = x^5 + 3x - 2$, we get $4 = x^5 + 3x$, so $x = 1$ (when $y = 2$) by observation. Then

$$(f^{-1})'(y) = \frac{1}{5x^4 + 3} \quad \text{and} \quad (f^{-1})'(2) = \frac{1}{5 \cdot 1 + 3} = \frac{1}{8}.$$

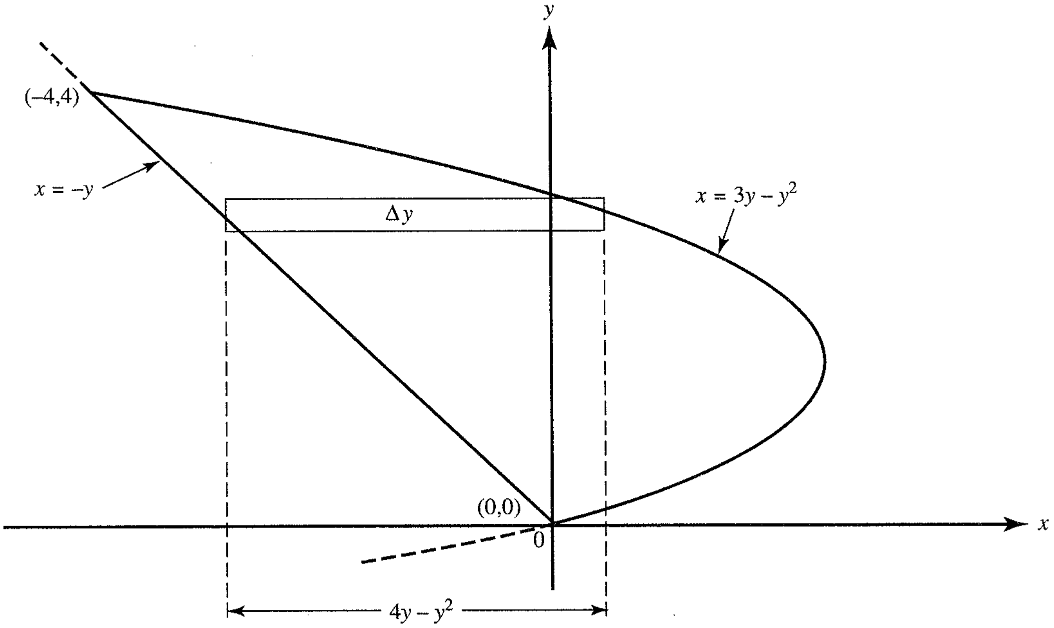
26. A. $\int_1^e \frac{\ln^3 x}{x}\,dx = \frac{1}{4}\ln^4 x \Big|_1^e = \frac{1}{4}(\ln^4 e - 0) = \frac{1}{4}$.

27. E. $\ln(4 + x^2) = \ln(4 + (-x)^2)$; $y' = \frac{2x}{4 + x^2}$; $y'' = \frac{-2(x^2 - 4)}{(4 + x^2)^2}$.

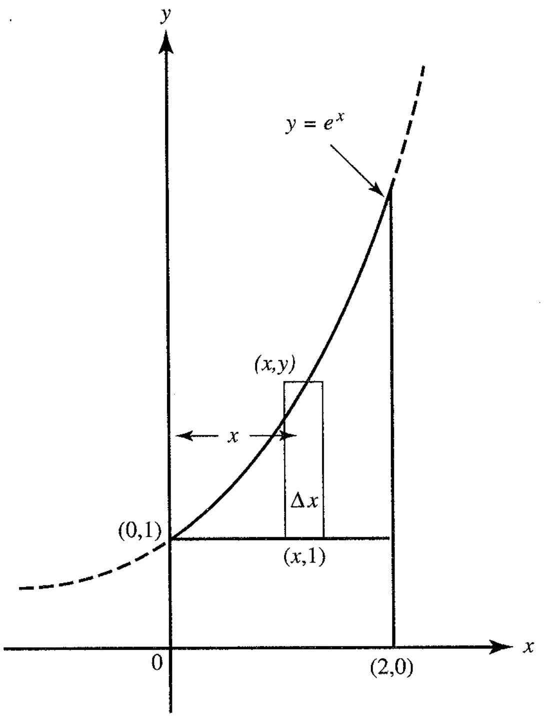
28. A. $f(x) = \frac{d}{dx}(x \sin \pi x) = \pi x \cos \pi x + \sin \pi x$.

Part B

29. B. $A = \int_0^4 [3y - y^2 - (-y)]\, dy = \int_0^4 (4y - y^2)\, dy$

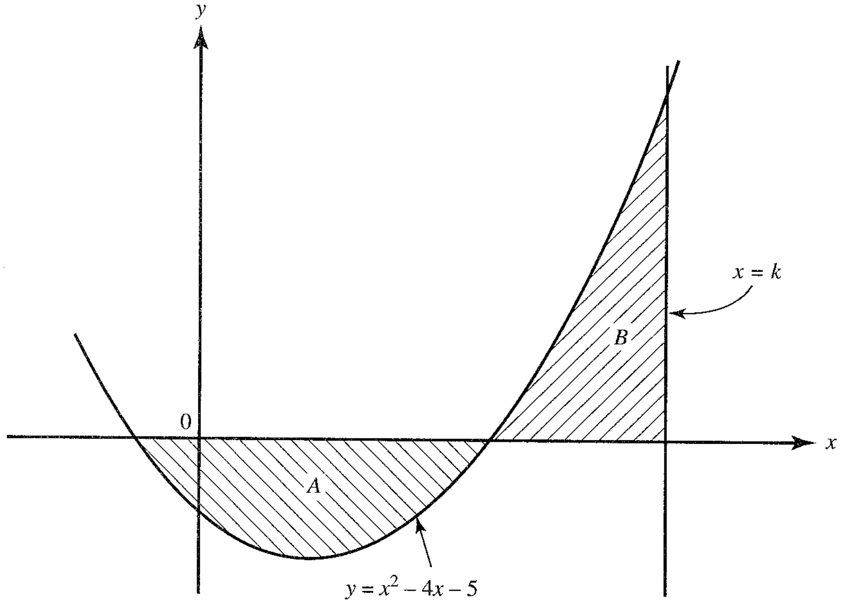


30. C. About the x-axis. Washer. $\Delta V = \pi(y^2 - 1^2)\, \Delta x$. $V = \pi \int_0^2 (e^{2x} - 1)\, dx$.



31. E. Since $v = \frac{ds}{dt} = 4s$, we have $\frac{ds}{s} = 4dt$. So $\ln s = 4t + C$, where, using $s(0) = 3$, $C = \ln 3$. Then, since $s = 3e^{4t}$, $s\left(\frac{1}{2}\right) = 3e^2$.

32. D.



(This figure is not drawn to scale.)

The roots of $f(x) = x^2 - 4x - 5 = (x - 5)(x + 1)$ are $x = -1$ and 5. Since areas A and B are equal, therefore $\int_{-1}^{k} f(x)\,dx = 0$. Thus,

$$\left(\frac{x^3}{3} - 2x^2 - 5x\right)\Bigg|_{-1}^{k} = \left(\frac{k^3}{3} - 2k^2 - 5k\right) - \left(-\frac{1}{3} - 2 + 5\right)$$
$$= \frac{k^3}{3} - 2k^2 - 5k - \frac{8}{3} = 0$$

Using 6 as a guess for the root k, enter

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solve(X³/3-2X²-5X-8/3,X,6)
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This yields $k = 8$.

33. D. If N is the number of bacteria at time t, then $N = 200e^{kt}$. We are told that $3 = e^{10k}$. When $t = 24$, $N = 200e^{24k}$. Therefore $N = 200(e^{10k})^{2.4} = 200(3)^{2.4} \simeq 2793$ bacteria.

34. C. Since $t = \frac{x + 1}{2}$, $dt = \frac{1}{2}dx$. For $x = 2t - 1$, $t = 3$ yields $x = 5$ and $t = 5$ yields $x = 9$.

35. C. Using implicit differentiation on the equation

$$x^3 + xy - y^2 = 10 \qquad (1)$$

yields

$$3x^2 + x\frac{dy}{dx} + y - 2y\frac{dy}{dx} = 0$$

$$3x^2 + y = (2y - x)\frac{dy}{dx}$$

and

$$\frac{dy}{dx} = \frac{3x^2 + y}{2y - x}$$

The tangent is vertical when $\frac{dy}{dx}$ is undefined; that is, when $2y - x = 0$. Replacing y by $\frac{x}{2}$ in (1), we have

$$x^3 + \frac{x^2}{2} - \frac{x^2}{4} = 10$$

or

$$4x^3 + x^2 = 40$$

Let $y_1 = 4x^3 + x^2 - 40$. By inspection of the equation $y_1 = f(x) = 0$, we see that there is a root near $x = 2$. Let Y_1=4X^3+X^2-40. To find the root, evaluate

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solve(Y1,X,2)
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The answer is 2.074.

36. D. $G'(x) = f(3x - 1) \cdot 3$.

37. B. Since f changes from positive to negative at $t = 3$, G' does also where $3x - 1 = 3$.

38. E. There are two problems to deal with in this question. The first is in entering the function correctly, the second in remembering to find the slope of the normal line (which is the negative reciprocal of the slope of the tangent). We first find $\frac{dy}{dx}$ in

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nDeriv((tan⁻¹(1/ln X))²,X,2)
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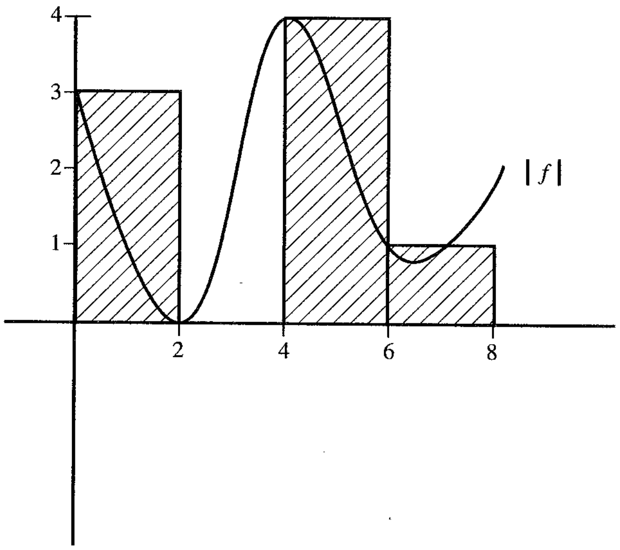
To obtain the reciprocal of $\frac{dy}{dx}$, we then raise the derivative above to the -1 power by keying in x^{-1}. This adds the exponent -1 after the last closing parenthesis. When we strike ENTER, we see $-1.5346. . .$ The answer we seek is therefore 1.535.

39. E. $2(3) + 2(0) + 2(4) + 2(1)$.

See the Figure.



40. E. $\frac{d}{dx}(f^2(x)) = 2f(x)f'(x)$,

$$\frac{d^2}{dx^2}(f^2(x)) = 2[f(x)f''(x) + f'(x)f'(x)]$$

$$= 2[ff'' + (f')^2].$$

At $x = 3$; the answer is $2[2(-2) + 5^2] = 42$.

41. D. Counterexamples are, respectively: for (A), $f(x) = |x|$, $c = 0$; for (B), $f(x) = x^3$, $c = 0$; for (C), $f(x) = x^4$, $c = 0$; for (E), $f(x) = x^2$ on $(-1, 1)$.

42. D. $\frac{df}{dt} = (x^2 + 1)\frac{dx}{dt}$. Find x when $\frac{df}{dt} = 10\frac{dx}{dt}$.

$$10\frac{dx}{dt} = (x^2 + 1)\frac{dx}{dt}$$

implies that $x = 3$.

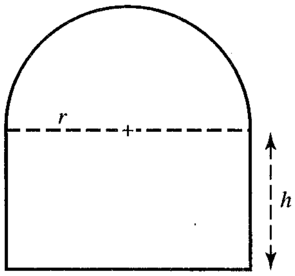
43. C. Let the circle have radius r and the rectangle height h. Then the perimeter $P = 2r + 2h + \frac{1}{2}(2\pi r)$, and the area $A = 2rh + \frac{1}{2}\pi r^2 = 100$. Solving for h in the latter yields

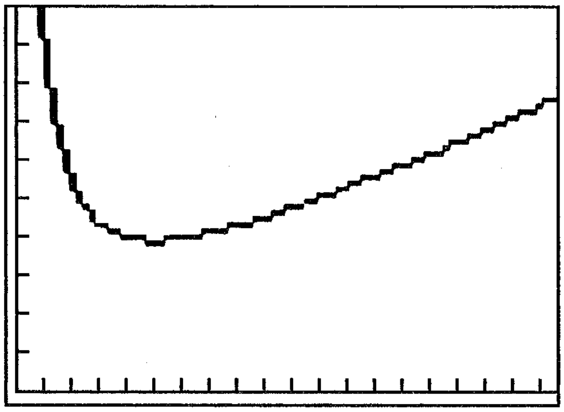
$$h = \frac{50}{r} - \frac{\pi r}{4}$$

Substituting for h in P, we have

$$P = 2r + 2\left(\frac{50}{r} - \frac{\pi r}{4}\right) + \pi r$$
$$= 2r + \frac{100}{r} + \frac{\pi r}{2}$$

Using X for r and Y_1 for P, we graph Y_1 in $[0, 20] \times [0, 100]$. The calculator's [minimum] option yields $Y_1 = 37.793$ as the least perimeter.





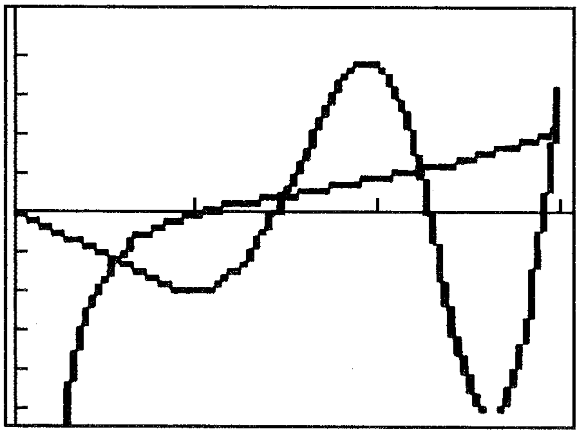
44. E. The velocity functions are

$$v_1 = -2t \sin(t^2 + 1)$$

and

$$v_2 = \frac{2t(e^t) - 2e^t}{(2t)^2} = \frac{e^t(t-1)}{2t^2}$$

Let X be t, let $Y_1 = -2X\sin(X^2 + 1)$ and $Y_2 = (e^X)(X - 1)/2X^2$. Graph Y_1 and Y_2 in $[0, 3] \times [-5, 5]$. The graphs intersect four times during the first three seconds.



45. B. $\dfrac{\int_{100}^{200} 50e^{-0.015t}dt}{100} \simeq 5.78 \text{ lb.}$