AB Practice Examination 3

Section I

Part A†

(See instructions, page 467. Answers begin on page 506.)

1. $\lim_{x \to 2} \dfrac{x^2 - 2}{4 - x^2}$ is

(A) -2 **(B)** -1 **(C)** $-\dfrac{1}{2}$ **(D)** 0 **(E)** nonexistent

2. $\lim_{x \to \infty} \dfrac{\sqrt{x} - 4}{4 - 3\sqrt{x}}$ is

(A) $-\dfrac{1}{3}$ **(B)** -1 **(C)** ∞ **(D)** 0 **(E)** $\dfrac{1}{3}$

3. If $y = \dfrac{e^{\ln u}}{u}$, then $\dfrac{dy}{du}$ equals

(A) $\dfrac{e^{\ln u}}{u^2}$ **(B)** $e^{\ln u}$ **(C)** $\dfrac{2e^{\ln u}}{u^2}$ **(D)** 1 **(E)** 0

4. Using the local linearization of $f(x) = \sqrt{9 + \sin(2x)}$ near 0, an estimate of $f(0.06)$ is

(A) 0.02 **(B)** 2.98 **(C)** 3.01 **(D)** 3.02 **(E)** 3.03

5. Air is escaping from a balloon at a rate of $R(t) = \dfrac{60}{1 + t^2}$ ft^3/min, where t is measured in minutes. How much (in ft^3) escapes during the first minute?

(A) 15 **(B)** 15π **(C)** 30 **(D)** 30π **(E)** $30 \ln 2$

6. If $y = \sin^3(1 - 2x)$, then $\dfrac{dy}{dx}$ is

(A) $3 \sin^2(1 - 2x)$ **(B)** $-2 \cos^3(1 - 2x)$ **(C)** $-6 \sin^2(1 - 2x)$
(D) $-6 \sin^2(1 - 2x) \cos(1 - 2x)$ **(E)** $-6 \cos^2(1 - 2x)$

†Beginning in May 1998, 55 minutes will be allowed for Part A.

7. If $y = x^2e^{1/x}$ $(x \neq 0)$, then $\dfrac{dy}{dx}$ is

(A) $xe^{1/x}(x + 2)$ **(B)** $e^{1/x}(2x - 1)$ **(C)** $\dfrac{-2e^{1/x}}{x}$ **(D)** $e^{-x}(2x - x^2)$
(E) none of these

8. A point moves along the curve $y = x^2 + 1$ so that the x-coordinate is increasing at the constant rate of $\dfrac{3}{2}$ units per second. The rate, in units per second, at which the distance from the origin is changing when the point has coordinates $(1, 2)$ is equal to

(A) $\dfrac{7\sqrt{5}}{10}$ **(B)** $\dfrac{3\sqrt{5}}{2}$ **(C)** $3\sqrt{5}$ **(D)** $\dfrac{15}{2}$ **(E)** $\sqrt{5}$

9. $\lim\limits_{h \to 0} \dfrac{\sqrt{25 + h} - 5}{h}$

(A) $= 0$ **(B)** $= \dfrac{1}{10}$ **(C)** $= 1$ **(D)** $= 10$ **(E)** does not exist

10. The base of a solid is the first-quadrant region bounded by $y = \sqrt[4]{1 - x^2}$. Each cross-section perpendicular to the x-axis is a square with one edge in the xy-plane. The volume of the solid is

(A) $\dfrac{2}{3}$ **(B)** $\dfrac{\pi}{4}$ **(C)** 1 **(D)** $\dfrac{\pi}{2}$ **(E)** π

11. $\displaystyle\int \frac{x\,dx}{\sqrt{9 - x^2}}$ equals

(A) $-\dfrac{1}{2}\ln\sqrt{9 - x^2} + C$ **(B)** $\sin^{-1}\dfrac{x}{3} + C$ **(C)** $-\sqrt{9 - x^2} + C$
(D) $-\dfrac{1}{4}\sqrt{9 - x^2} + C$ **(E)** $2\sqrt{9 - x^2} + C$

12. $\displaystyle\int \frac{(y - 1)^2}{2y}dy$ equals

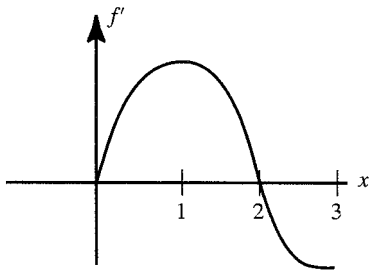
(A) $\dfrac{y^2}{4} - y + \dfrac{1}{2}\ln|y| + C$ **(B)** $y^2 - y + \ln|2y| + C$
(C) $y^2 - 4y + \dfrac{1}{2}\ln|2y| + C$ **(D)** $\dfrac{(y - 1)^3}{3y^2} + C$ **(E)** $\dfrac{1}{2} - \dfrac{1}{2y^2} + C$

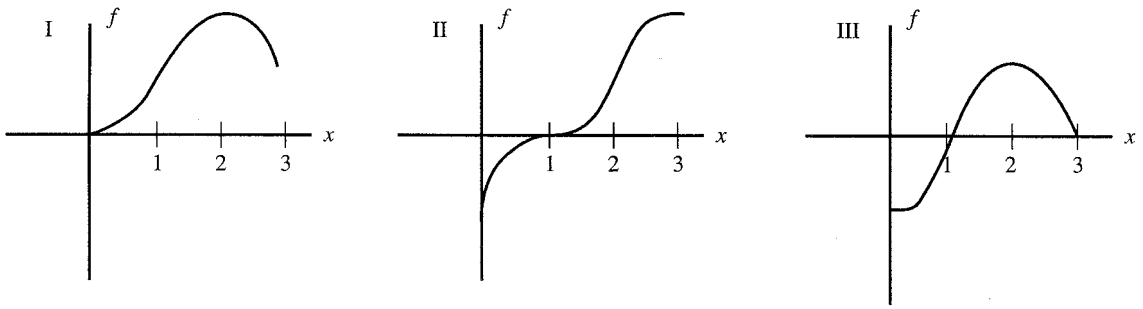
13. $\displaystyle\int_{\pi/6}^{\pi/2} \cot x\,dx$ equals

(A) $\ln\dfrac{1}{2}$ **(B)** $\ln 2$ **(C)** $-\ln(2 - \sqrt{3})$ **(D)** $\ln(\sqrt{3} - 1)$
(E) none of these

14. Given f' as graphed, which could be a graph of f?

(A) I only **(B)** II only
(C) III only **(D)** I and III
(E) none of these

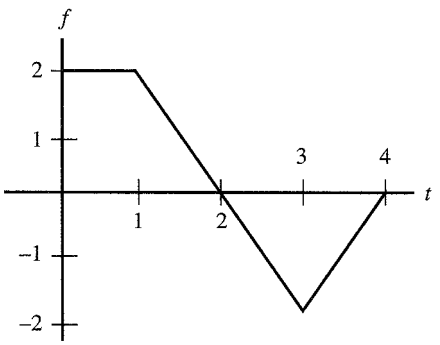




15. Women have recently begun competing in marathons. At first, their times for the 26-mile event dropped rapidly, but of late, the times have been declining at a much slower rate. Let $M(t)$ be the curve which best represents winning marathon times in year t. Which of the following is negative?

I. $M(t)$ **II.** $M'(t)$ **III.** $M''(t)$

(A) I only **(B)** II only **(C)** III only **(D)** II and III
(E) none of these



16. The graph of f is shown. Let $G(x) = \int_0^x f(t)\,dt$ and $H(x) = \int_2^x f(t)\,dt$. Which of the following is true?

(A) $G(x) = H(x)$ **(B)** $G'(x) = H'(x + 2)$ **(C)** $G(x) = H(x + 2)$
(D) $G(x) = H(x) - 2$ **(E)** $G(x) = H(x) + 3$

17. The minimum value of $f(x) = x^2 + \frac{2}{x}$ on the interval $\frac{1}{2} \leqq x \leqq 2$ is

(A) $\frac{1}{2}$ **(B)** 1 **(C)** 3 **(D)** $4\frac{1}{2}$ **(E)** 5

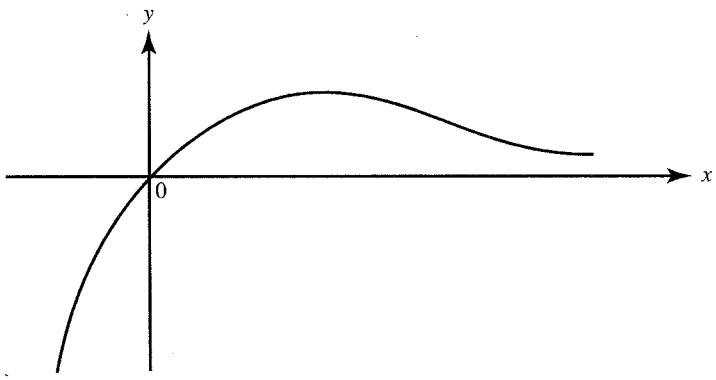
18. A particle moves along a line so that its acceleration, a, at time t is $a = -t^2$. If the particle is at the origin when $t = 0$ and 3 units to the right of the origin when $t = 1$, then its velocity at $t = 0$ is

(A) 0 **(B)** $\frac{1}{12}$ **(C)** $2\frac{11}{12}$ **(D)** $\frac{37}{12}$ **(E)** none of these

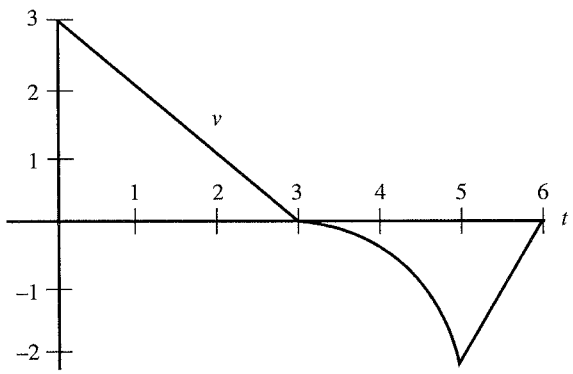
19. Which of the following functions could have the graph sketched below?

(A) $f(x) = xe^x$ **(B)** $f(x) = xe^{-x}$ **(C)** $f(x) = \frac{e^x}{x}$ **(D)** $f(x) = \frac{x}{x^2 + 1}$

(E) $f(x) = \frac{x^2}{x^3 + 1}$



This graph, consisting of two line segments and a quarter circle, is for Questions 20–22. It shows the velocity of an object during a 6-second interval.



20. For how many values of t is the acceleration undefined?

(A) none **(B)** one **(C)** two **(D)** three **(E)** four

21. During what time interval is the speed increasing?

(A) $0 < t < 3$ **(B)** $3 < t < 5$ **(C)** $3 < t < 6$ **(D)** $5 < t < 6$

(E) never

22. What is the average acceleration during the first five seconds (in units/sec^2)?

(A) $-\frac{5}{2}$ **(B)** -1 **(C)** $-\frac{1}{5}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{1}{2}$

23. The curve of $y = \dfrac{2x^2}{4 - x^2}$ has

(A) two horizontal asymptotes
(B) two horizontal asymptotes and one vertical asymptote
(C) two vertical but no horizontal asymptotes
(D) one horizontal and one vertical asymptote
(E) one horizontal and two vertical asymptotes

24. Suppose

$$f(x) = \begin{cases} x^2 & \text{if } x < -2, \\ 4 & \text{if } -2 < x \leqq 1, \\ 6 - x & \text{if } x > 1. \end{cases}$$

Which statement is true?

(A) f is discontinuous only at $x = -2$. **(B)** f is discontinuous only at $x = 1$.
(C) If $f(-2)$ is defined to be 4, then f will be continuous everywhere.
(D) f is continuous everywhere. **(E)** f is discontinuous at $x = -2$ and at $x = 1$.

25. Let $f(x) = x^5 + 3x - 2$, and let f^{-1} denote the inverse of f. Then $(f^{-1})'(2)$ equals

(A) $\dfrac{1}{83}$ **(B)** $\dfrac{1}{8}$ **(C)** 1 **(D)** 8 **(E)** 83

26. $\displaystyle\int_1^e \frac{\ln^3 x}{x}\,dx =$

(A) $\dfrac{1}{4}$ **(B)** $\dfrac{1}{4}e$ **(C)** $\dfrac{1}{4}(e - 1)$ **(D)** $\dfrac{e^4}{4}$ **(E)** $\dfrac{e^4 - 1}{4}$

27. Which of the following statements are true about the graph of $y = \ln(4 + x^2)$?

I. It is symmetric to the y-axis.
II. It has a local minimum at $x = 0$.
III. It has inflection points at $x = \pm 2$.

(A) I only **(B)** II only **(C)** III only **(D)** I and II only
(E) I, II, and III

28. Let $\displaystyle\int_0^x f(t)\,dt = x \sin \pi x$. Then $f(3) =$

(A) -3π **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 3π

Part B†

(See instructions, page 471. Answers begin on page 509.)

29. The area bounded by the curve $x = 3y - y^2$ and the line $x = -y$ is represented by

(A) $\displaystyle\int_0^4 (2y - y^2)\,dy$ **(B)** $\displaystyle\int_0^4 (4y - y^2)\,dy$ **(C)** $\displaystyle\int_0^3 (3y - y^2)\,dy + \int_0^4 y\,dy$

(D) $\displaystyle\int_0^4 (y^2 - 4y)\,dy$ **(E)** $\displaystyle\int_0^3 (2y - y^2)\,dy$

†Beginning in May 1998, 50 minutes will be allowed for Part B.

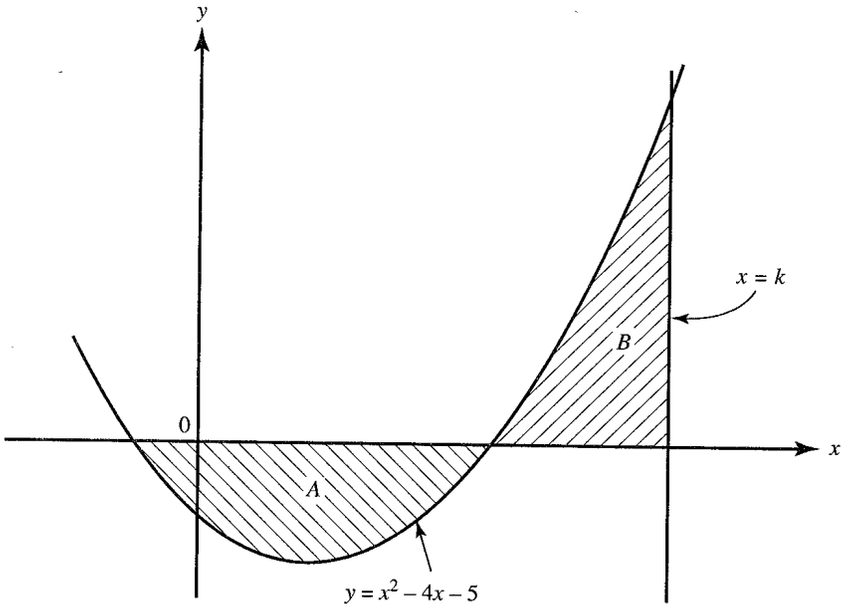
30. The region bounded by $y = e^x$, $y = 1$, and $x = 2$ is rotated about the x-axis. The volume of the solid generated is given by the integral:

(A) $\pi\int_0^2 e^{2x}\,dx$ **(B)** $2\pi\int_1^{e^2}(2 - \ln y)(y - 1)\,dy$ **(C)** $\pi\int_0^2 (e^{2x} - 1)\,dx$

(D) $2\pi\int_0^{e^2} y\,(2 - \ln y)\,dy$ **(E)** $\pi\int_0^2 (e^x - 1)^2\,dx$

31. A particle moves on a straight line so that its velocity at time t is given by $v = 4s$, where s is its distance from the origin. If $s = 3$ when $t = 0$, then, when $t = \frac{1}{2}$, s equals

(A) $1 + e^2$ **(B)** $2e^3$ **(C)** e^2 **(D)** $2 + e^2$ **(E)** $3e^2$



(This figure is not drawn to scale.)

32. The sketch shows the graphs of $f(x) = x^2 - 4x - 5$ and the line $x = k$. The regions labeled A and B have equal areas if $k =$

(A) 5 **(B)** 7.766 **(C)** 7.899 **(D)** 8 **(E)** 11

33. Bacteria in a culture increase at a rate proportional to the number present. An initial population of 200 triples in 10 hours. If this pattern of increase continues unabated, then the approximate number of bacteria after one full day is

(A) 1160 **(B)** 1440 **(C)** 2408 **(D)** 2793 **(E)** 8380

34. Using the substitution $x = 2t - 1$, the definite integral $\int_3^5 t\sqrt{2t - 1}\,dt$ may be expressed in the form $k\int_a^b (x + 1)\sqrt{x}\,dx$, where $\{k, a, b\} =$

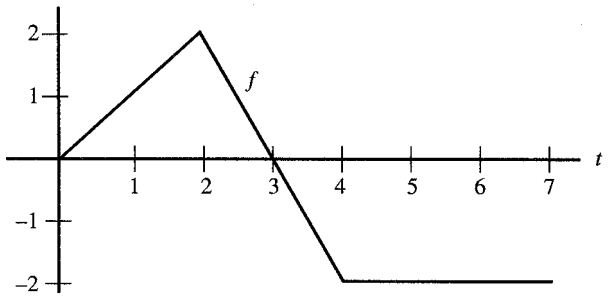
(A) $\left\{\frac{1}{4}, 2, 3\right\}$ **(B)** $\left\{\frac{1}{4}, 3, 5\right\}$ **(C)** $\left\{\frac{1}{4}, 5, 9\right\}$ **(D)** $\left\{\frac{1}{2}, 2, 3\right\}$

(E) $\left\{\frac{1}{2}, 5, 9\right\}$

35. The curve defined by $x^3 + xy - y^2 = 10$ has a vertical tangent line when $x =$

(A) 0 or $-\frac{1}{3}$ **(B)** 1.037 **(C)** 2.074 **(D)** 2.096 **(E)** 2.154

The graph of f shown on [0,7] is for Questions 36 and 37. Let $G(x) = \int_2^{3x-1} f(t)\,dt$.



36. $G'(1)$ is

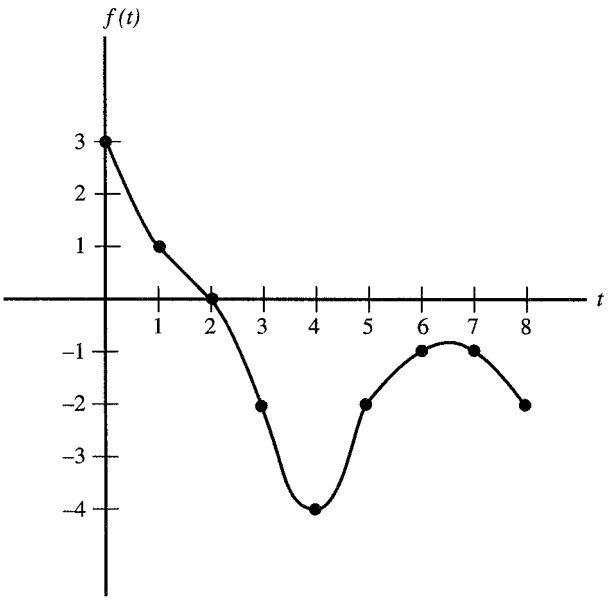
(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** undefined

37. G has a local maximum at $x =$

(A) 1 **(B)** $\frac{4}{3}$ **(C)** 2 **(D)** 5 **(E)** 8

38. The slope of the line normal to $y = (\text{arc cot } (\ln x))^2$ at $x = 2$ is

(A) −2.443 **(B)** −1.535 **(C)** −0.652 **(D)** 0.652 **(E)** 1.535



39. Using the left rectangular method, and 4 subintervals, estimate $\int_0^8 |f(t)|\,dt$, where f is the function graphed.

(A) 4 **(B)** 5 **(C)** 8 **(D)** 15 **(E)** 16

40. Suppose $f(3) = 2$, $f'(3) = 5$, and $f''(3) = -2$. Then $\frac{d^2}{dx^2}(f^2(x))$ at $x = 3$ is equal to

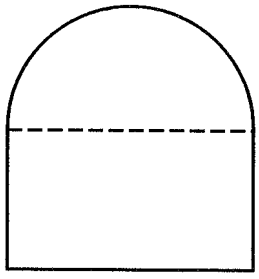
(A) -20 **(B)** 10 **(C)** 20 **(D)** 38 **(E)** 42

41. Which statement is true?

(A) If $f(x)$ is continuous at $x = c$, then $f'(c)$ exists.
(B) If $f'(c) = 0$, then f has a local maximum or minimum at $(c, f(c))$.
(C) If $f''(c) = 0$, then f has an inflection point at $(c, f(c))$.
(D) If f is differentiable at $x = c$, then f is continuous at $x = c$.
(E) If f is continuous on (a, b), then f attains a maximum value on (a, b).

42. Suppose $f(x) = \frac{1}{3}x^3 + x$, $x > 0$, and x is increasing. The value of x for which the rate of increase of f is 10 times that of x is

(A) 1 **(B)** 2 **(C)** $\sqrt[3]{10}$ **(D)** 3 **(E)** $\sqrt{10}$



43. The figure above consists of a rectangle capped by a semicircle. Its area is 100 yd^2. The minimum perimeter of the figure is

(A) 10.584 yd **(B)** 28.284 yd **(C)** 37.793 yd **(D)** 38.721 yd
(E) 51.820 yd

44. Two objects in motion from $t = 0$ to $t = 3$ seconds have positions, $x_1(t) = \cos(t^2 + 1)$ and $x_2(t) = \frac{e^t}{2t}$ respectively. How many times during the three seconds do the objects have the same velocity?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

45. After t years, $50e^{-0.015t}$ pounds of a deposit of a radioactive substance remains. The average amount per year *not* lost by radioactive decay during the second hundred years is

(A) 2.9 lb **(B)** 5.8 lb **(C)** 7.4 ln **(D)** 11.1 lb **(E)** none of these

Answers to AB Practice Examination 3: Section I

1.	E	**10.**	B	**19.**	B	**28.**	A	**37.**	B
2.	A	**11.**	C	**20.**	C	**29.**	B	**38.**	E
3.	E	**12.**	A	**21.**	B	**30.**	C	**39.**	E
4.	D	**13.**	B	**22.**	B	**31.**	E	**40.**	E
5.	B	**14.**	D	**23.**	E	**32.**	D	**41.**	D
6.	D	**15.**	B	**24.**	E	**33.**	D	**42.**	D
7.	B	**16.**	E	**25.**	B	**34.**	C	**43.**	C
8.	B	**17.**	C	**26.**	A	**35.**	C	**44.**	E
9.	B	**18.**	D	**27.**	E	**36.**	D	**45.**	B