Answers to AB Practice Examination 1: Section I

1.	C	**10.**	D	**19.**	B	**28.**	C	**37.**	A
2.	D	**11.**	E	**20.**	A	**29.**	B	**38.**	D
3.	A	**12.**	D	**21.**	D	**30.**	C	**39.**	E
4.	D	**13.**	E	**22.**	C	**31.**	D	**40.**	E
5.	E	**14.**	D	**23.**	C	**32.**	D	**41.**	C
6.	D	**15.**	B	**24.**	B	**33.**	B	**42.**	E
7.	E	**16.**	D	**25.**	D	**34.**	B	**43.**	D
8.	B	**17.**	E	**26.**	A	**35.**	A	**44.**	C
9.	C	**18.**	D	**27.**	E	**36.**	C	**45.**	D

Part A

1. C. Use the Rational Function Theorem (page 30).

2. D. $\lim\limits_{h\to 0}\dfrac{\cos\left(\dfrac{\pi}{2}+h\right)}{h}=\lim\limits_{h\to 0}\dfrac{-\sin h}{h}=-1.$

3. A. Although $f'(2)=f'(1)=0, f'(x)$ changes sign only as x increases through 1, and in this case $f'(x)$ changes from negative to positive.

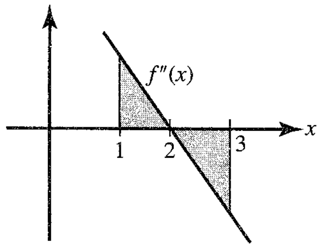
4. D. $F'(x)=\dfrac{10}{1+e^x}>0$, and $F''(x)=\dfrac{-10e^x}{(1+e^x)}<0.$

5. E. $\dfrac{f(1.04)-f(1)}{1.04-1}=\dfrac{0.96}{0.04}.$

6. D. The graph must look like one of these two:

7. E. $f'(x)=3\cos x\cos 3x-\sin x\sin 3x.$

8. B. Evaluate $\dfrac{1}{2}\ln(x^2+1)\Big|_0^1.$



9. C. Let $f'(x) = \int_1^x f''(t)\, dt$. Then f' increases for $1 << x < 2$, then begins to decrease. The area below the x-axis, from 2 to 3, is equal in magnitude to that above the x-axis.

10. D. $P'(x) = 2g(x) \cdot g'(x)$.

11. E. If $f(H(x)) = x$, then $f'(H(x)) \cdot H'(x) = 1$. Therefore $H'(3) = \frac{1}{f'(H(3))} = \frac{1}{f'(2)} = 1$. (Note that $H(3) = f^{-1}(3) = 2$.)

12. D. Note that the domain of y is all x such that $|x| \leqq 1$ and that the graph is symmetric to the origin. The area is given by

$$2\int_0^1 x\sqrt{1 - x^2}\, dx.$$

13. E. Since

$$y' = 2(x - 3)^{-2} \text{ and } y'' = -4(x - 3)^{-3} = \frac{-4}{(x - 3)^3},$$

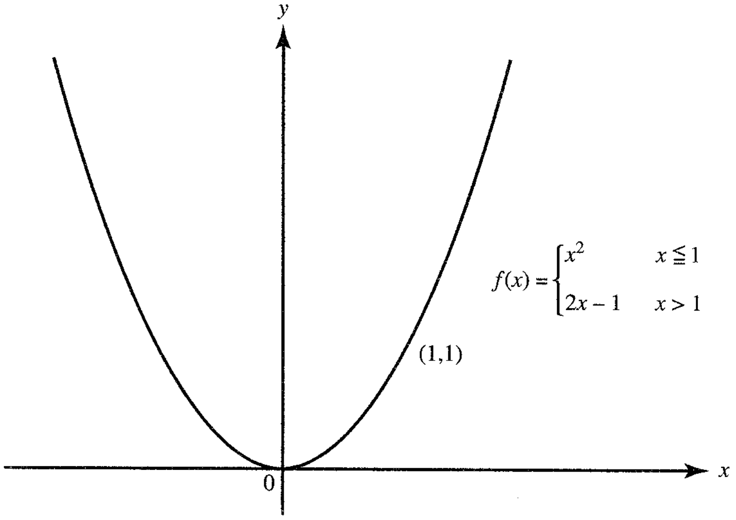
we see that y'' is positive when $x < 3$.

14. D. Draw a figure and let (x, y) be the point in the first quadrant where the line parallel to the x-axis meets the parabola. The area of the triangle is given by $A = xy = x(27 - x^2)$. Show that A is a maximum for $x = 3$.

15. B. $\frac{1}{\pi/3}\int_0^{\pi/3} \tan x\, dx = \frac{3}{\pi}\Big[-\ln \cos x\Big]_0^{\pi/3} = \frac{3}{\pi}\left(-\ln \frac{1}{2}\right)$.

16. D. If $x = 2t + 1$, then $t = \frac{x - 1}{2}$, so $dt = \frac{1}{2}\, dx$. When $t = 0$, $x = 1$; when $t = 3$, $x = 7$.

17. E. As the water gets deeper, the rate of change of depth decreases: $\frac{d^2h}{dt^2} < 0$.



18. D. The graph of f is shown in the figure; f is defined and continuous at all x, including $x = 1$. Since

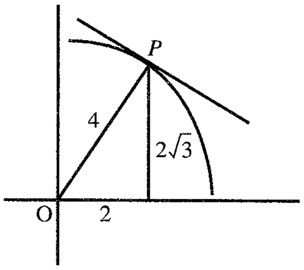
$$\lim_{x \to 1^-} f'(x) = 2 = \lim_{x \to 1^+} f'(x),$$

$f'(1)$ exists and is equal to 2.

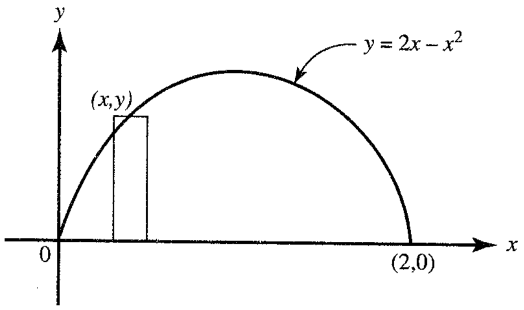
19. B. Since $|x - 2| = 2 - x$ if $x < 2$, the limit as $x \to 2^-$ is $\frac{2 - x}{x - 2} = -1$.

20. A. The average speed $= \dfrac{\text{distance covered in 6 seconds}}{\text{time elapsed}}$

$$= \frac{\frac{1}{4}\pi(4^2) + \frac{1}{2}(1 \cdot 2) + 1 \cdot 2}{6}$$



21. D. Acceleration is the slope of the velocity curve; since the slope of the radius $OP = \sqrt{3}$, the slope of the tangent is $-\frac{1}{\sqrt{3}}$.



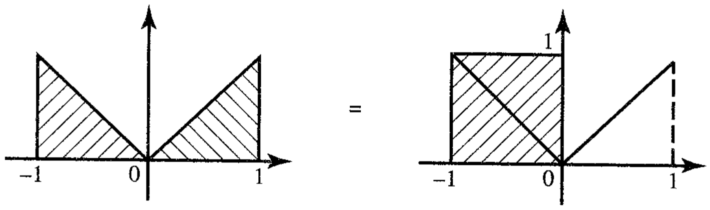
22. C. About the x-axis. DISK.

$$\Delta V = \pi R^2 H = \pi y^2 \Delta x$$

$$V = \pi \int_0^2 (2x - x^2)^2 dx$$

23. C. If P denotes the initial value of the account and A is the value at time t, then $A = Pe^{0.06t}$. We seek t when $A = 3P$: $3 = e^{0.06t}$ yields $t = (\ln 3)/0.06 > 18$ years.

24. B.

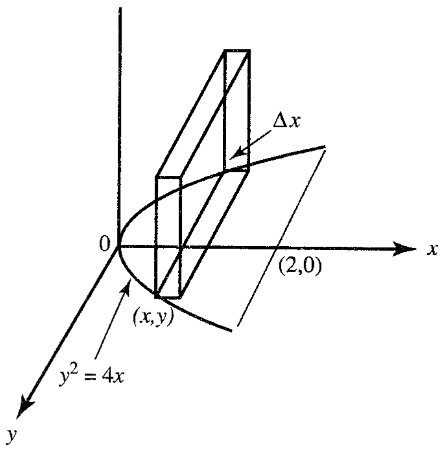


25. D. $\int_a^x g(t)\,dt - \int_b^x g(t)\,dt = \int_a^x g(t)\,dt + \int_x^b g(t)\,dt = \int_a^b g(t)\,dt$

26. A. Separate to get $\frac{dy}{y^2} = 2x\,dx$, $-\frac{1}{y} = x^2 + C$. Since $-(-1) = 1 + C$ implies that $C = 0$, the solution is

$$-\frac{1}{y} = x^2 \quad \text{or} \quad y = -\frac{1}{x^2}.$$

27. E.



$$\Delta V = (2y)^2 \Delta x;$$

$$V = 4\int_0^2 y^2\,dx$$

$$= 4\int_0^2 4x\,dx$$

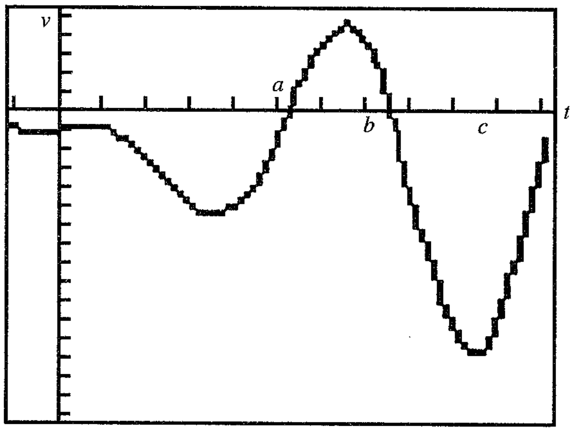
$$= 32.$$

28. C. Note that $\lim_{x\to\infty} \frac{x^2}{e^x} = 0$, $\lim_{x\to-\infty} \frac{x^2}{e^x} = \infty$, and $\frac{x^2}{e^x} \geq 0$ for all x.

Part B

29. B. At $x = 3$, the equation of the tangent line is $y - 8 = -4(x - 3)$; so $f(x) \simeq -4(x - 3) + 8$. $f(3.02) \simeq -4(0.02) + 8$.

30. C.



Use Y_1 for velocity and X for time; then

$$Y_1 = X\cos X - \ln(X + 2)$$

We graph Y_1 in $[-1, 11] \times [-15, 5]$. The object reverses direction when the velocity changes sign; that is, when the graph crosses the X-axis. There are two such reversals—at $X = a$ and at $X = b$.

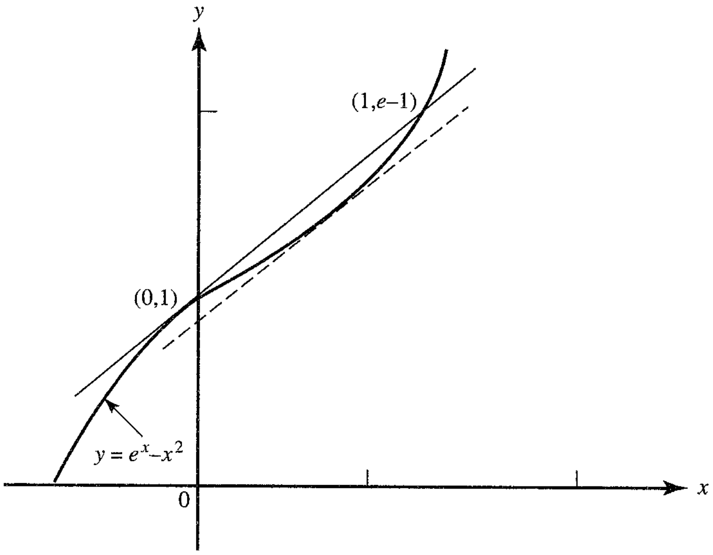
31. D. The sign diagram

f		inc		inc		dec
	0		2		4	5
f'		+		+		−

shows that f has a maximum at $x = 4$ and a minimum at the endpoints.

32. D. Since f' decreases, increases, then decreases, f'' changes from negative to positive, then back to negative. These sign changes occur at $x = 2$ and $x = 3$.

33. B.



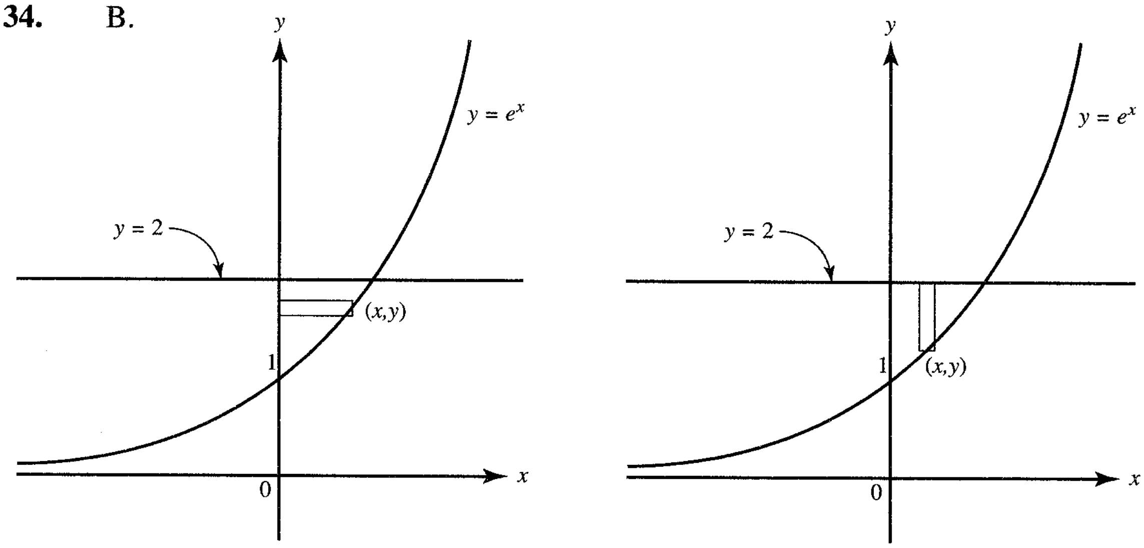
On the curve of $f(x) = e^x - x^2$, the two points are (0,1) and $(1, e - 1)$. The slope of the secant line is $m = \frac{\Delta y}{\Delta x} = \frac{e - 2}{1} - e - 2$. We seek c in [0,1] such that $f'(c) = e - 2$, or $f'(c) - (e - 2) = 0$. Since $f'(x) = e^x - 2x$, c can be calculated from

solve (e^X − 2X − (e − 2), X, .5)

The answer is 0.351.

If we let Y_1, = e^x − 2X − e + 2 and graph Y_1, then we can also locate c by using the [root] option. The x-coordinate of the root, shown on the screen, is 0.351. (Verify this alternative solution.)

34. B.



Using disks, $\Delta V = \pi R^2 H = \pi(\ln y)^2 \Delta y$. Note that the limits of the definite integral are 1 and 2. With change of variables for the calculator, we evaluate the expression

πfnInt((ln X)^2, X,1 ,2) = 0.592.

Using shells, $\Delta V = 2\pi RHT = 2\pi x(2 - e^x)\Delta x$. Here, the upper limit of integration is the value of x for which $e^x = 2$, namely ln 2. We now calculate

πfnInt(X(2-e^x), X, 0, ln 2)

Again, we get 0.592. (Volume by shells is not a topic in the May '98, May '99 AP Course Description.)

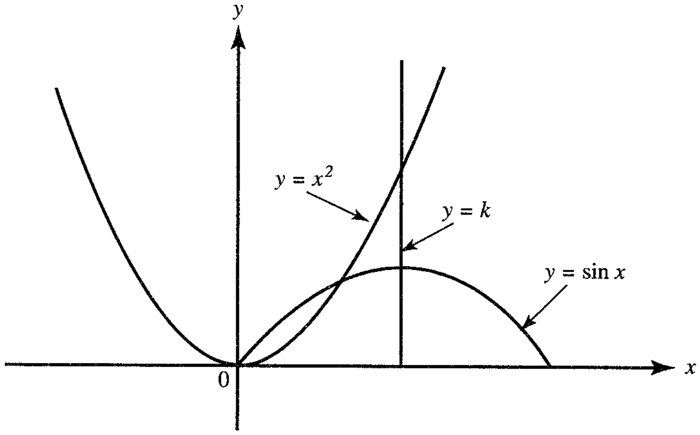
35. A. Differentiating implicitly yields $2xyy' + y^2 - 2y' + 12y^2y' = 0$. When $y = 1$, $x = 4$. Substitute to find y'.

36. C. $v = 3t^2 + 1$ and $s = t^3 + t + 3$.

37. A. Let $u = x^2$. Then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{df}{du} \cdot f'(u)\frac{du}{dx} = \sqrt{5u - 1} \cdot 2x = 2x\sqrt{5x^2 - 1}.$$

38. D.



We want $\int_0^k \sin x\, dx$ to equal $\int_0^k x^2\, dx$. We have then

$$-\cos x\Big|_0^k = \frac{x^3}{3}\Big|_0^k$$

$$-\cos k - (-\cos 0) = \frac{k^3}{3} - 0$$

$$-\cos k + 1 = \frac{k^3}{3}$$

$$0 = \frac{k^3}{3} + \cos k - 1$$

To find k we evaluate

solve $(X^3/3 + \cos X - 1, X, 1)$

The answer is 1.300.

39. E. $[\cos(x^2)]' = -\sin(x^2) \cdot 2x$. We cannot create the missing factor $2x$ through introduction of constants alone.

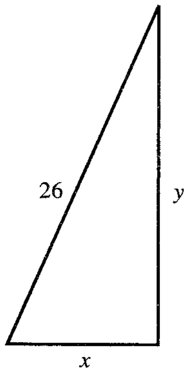
40. E. Use $A(t) = A_0e^{kt}$, where the initial amount A_0 is 50. Then $A(t) = 50e^{kt}$. Since 45 grams remain after 9 days, we have $45 = 50e^{k \cdot 9}$, which yields $k = \frac{\ln 0.9}{9}$.

To find t when 20 grams remain, we must solve

$$20 = 50e^{\left(\frac{\ln 0.9}{9}\right) \cdot t}$$

Thus,

$$t = \frac{9 \ln 0.4}{\ln 0.9} = 78.3$$



41. C. See the figure. Since $x^2 + y^2 = 26^2$ and since it is given that $\frac{dx}{dt} = 3$, it follows that

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad \text{and} \quad \frac{dy}{dt} = -\frac{x}{y}(3)$$

at any time t. When $x = 10$, then $y = 24$ and $\frac{dy}{dt} = \frac{-5}{4}$.

42. E. Let $u = 2x$ and note that $F'(u) = \frac{1}{1 - u^3}$. Then

$$F'(x) = F'(u)u'(x) = 2F'(u) = 2 \cdot \frac{1}{1 - (2x)^3}.$$

43. D. $v(5) - v(0) = \int_0^5 a(t)\,dt = -\frac{1}{4}\pi \cdot 2^2 + \frac{1}{2}(3)(2) = -\pi + 3$. Since $v(5) = 0$, $-v(0) = -\pi + 3$; so $v(0) = \pi - 3$.

44. C. $\int_0^{24} 5 \arctan\left(\frac{t}{5}\right) dt = 124.102$

45. D. Let $y = (x^3 - 4x^2 + 8)e^{\cos(x^2)}$. The equation of the tangent at point $(2, y(2))$ is $y - y(2) = y'(2)(x - 2)$. Note that $y(2) = 0$. To find the y-intercept we let $x = 0$ and solve for y: $y = -2y'(2)$. With `X` for x and $\mathtt{Y}_1$ for y,

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-2nDeriv(Y1,X,2)
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yields 4.161.